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A

PRACTICAL

Treatise of

Arithmetick:

Plain SHEWING

Without the Help of a Master;

A Plain and Easie way of Working upon Whole Numbers and Fractions.

AS ALSO

Of readily managing the Rule of Three, and the other Rules depending thereupon.

By *Tho. York*.

Licensed and Entred according to Order. *R. Midgley*.

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James Brough Book
God Giue Him Grace
On It To Look And When
The Bel For Him Doth
Toul The Lord in Heaven
Receive His Soul
Amen 1707

James Brough his
Booke god giue him grace
on it to Look and when
The Bel for him doth
toul the liuing Lord recei
his soul Amen 1705
James Brough

The Preface to the Reader.

THat this preliminary discourse may not prove altogether unprofitable to the Reader, I shall wave all things seemingly much more really useless and unnecessary. Therefore to come presently to the main point, Arithmetick or (in plainer English) the Art of counting or (if you please) Cyphering, pretends to nought else but to instruct a man how by the help of two or more quantities or things given or known, he may come to the knowledge of something he before was altogether ignorant of. The method this ingenious Art prescribes (to effect this) is adding to and taking from, and really this is its ALL. Nor is this the whole of Arithmetick only, but even of the sublimest and most angelical of all Sciences, to wit Mathematical investigation or Art of discovering secrets in quantity. The truth of which assertion might easily be made appear, were

this

this a proper place for ſuch a buſineſs. As to the Art now under conſideration. If we have any two numbers given, and would know the ſum of them, or a number that's equal to both, it teaches us how to obtain our deſire, viz. *by adding them together. If two numbers are given, a greater and a leſs, and you deſire to know the exceſs of the greater above the leſs, the Rule pertaining to the third Problem puts you into a ready way to accompliſh your purpoſe,* viz. *by* Subſtraction. *Again two numbers being known and you would find out another number, between which and one of them there is the ſame ration, rate or proportion that there is between the other and unity, this Art helps you to a ready expedient,* viz. Multiplication, *which will do your buſineſs. Once more, any two numbers being aſſigned and a perſon is deſirous to meet with a number that bears the ſame relation (in reſpect of quantity) to unity, the greateſt of the known numbers does to the leſs, this Art furniſhes him with a medium whereby he may readily do this alſo,*

viz.

viz. Division. *Lastly, any three numbers being given, and a man wants a fourth, between which and the third known number, there shall be the same Ration that there is between the first and second: Arithmetick has in her well furnished Magazine, a tool (whose usefulness is beyond expression) by means of which he may easily and speedily fulfil his desire, and that is the* Golden Rule. *Now the performance of this Rule many times leads the* Practitioner *to broken numbers, or pieces of numbers, which as often happens out, as the lesser number that divides the greater proves to be no aliquot part of it, that is, does not exactly measure it. When therefore such numbers as these occur, Arithmetick teaches her Children how to manage them to the best advantage, and to handle them just as they did* Integers *or whole Numbers before. Thus having given thee (Reader) a short survey of this excellent Art, I come next to acquaint thee what is here done toward the fulfilling its principal drift and design, and to let thee know how far this*

Treatise may be serviceable to any one who (desirous of a competent skill therein) shall think it worth his while to spend sometime in the perusal of it. In order to this I must tell the Reader that these Rules are not new or lately invented, but hatch'd long since. However this much I dare say for them, that they are laid down in as advantagious a manner (to be understood) as need to be. And no wonder since they were moulded into the shape they are at present of by the acute wit and unwearied diligence of divers men of as great ability and skill in this kind of Learning as any this age has produced. The names of these Authors out of whose Elaborate writings this Treatise was extracted might here be put down, but I think 'twould little or nothing boot or benefit those who intend nothing but ordinary practise. For what would it avail a common Practiser to know the names of Books that are every where fraught with most profound notions, and very sublime speaculations, that are much above the reach of his apprehension and understanding?

ing? Moreover I here boldly aver that in this Treatise as little as 'tis there's never a useful Rule omitted: nor are the Rules here laid down abridged and curtailed, but out at full length, rather too long than otherwise. Besides the Reader will find several different wayes and ingenious ones too of performing Multiplication, Division and the Golden Rule, more perhaps than any Arithmetick yet published can furnish him with. The reason why this Treatise has not swelled into the same bulk with others that treat of the same subject, is because the compiler thereof suppos'd the great number of Examples that are in other mens writings, to take up a great deal of room in their Books to no purpose. The Rules being well weigh'd and throughly understood, the practical questions that fall under them are within ken and may easily be perceived by any one that is not bleareyed in his understanding, and may with a wet finger be dispatched. And really I have a better opinion of my Countrey-men, then to think them to be so blockish as that if a joynt of meat was neat-

ly Cooked, handsomely disht and set upon the Table, they wanted one to teach them how to feed themselves, or to put the meat into their mouths. To make a long declamation upon the usefulness of this Art were a very easie task, but in my opinion a very vain and idle one. For what Tradesman or Dealer is there but readily acknowledges Arithmetick to be mighty useful in all callings whatsoever. There's one thing behind which I must not forget and that is to tell the Reader that with very little pains he may read this Treatise through and understand it. The way is level and smooth, there's no knobs for him to stumble at, nor pits into which he may fall, no Thorns or Briers to catch hold of him throughout the Treatise. The language is plain and easie, here's no occasion at every turn to consult English Interpreters and Glossographics. This Art affects a plain dress, and mortally hates the quaint expressions suckt from Rhetorick, also the refined and abstracted notions of the Schools, and no less abominates the ridiculous criticisms so much doted upon by

Grammarians. The ſubject-matter of this Treatiſe lyes low, and is I am confident within the reach of any ordinary capacity. One thing more there is that mightily facilitates the underſtanding of it and that is the dependance one part has upon another, all the parts of it being as it were link'd together. For he that ſhall begin this Treatiſe, and go gently but withal heedfully on, will eaſily perceive that the underſtanding of one thing will lead him into the knowledge of the next, and this into the knowledge of what follows it, and ſo quite to the end of it. To what has been ſaid touching the little pains the Reader need take to underſtand what is contained in this Treatiſe may be reply'd, that never a Rule is here laid down whereby to guide the Learner whilſt employed about Diviſion how he may at firſt daſh light upon the right figure to be placed in the Quotient ſo as never to miſs. To this I anſwer that it muſt be confeſt, that no ſuch Rule is here to be found, nor do I believe in any other Treatiſe whatſoever, notwithſtanding I affirm that after

two or three tryals (especially if the Learner carefully mind what he is about) at the most 'tis ten to one but he meets with it. I add further that a little practise and experience will if not wholly at least wise so far take away this difficulty, that in a short time the pain or irksomeness arising from thence will scarce be felt and so not minded. That therefore the Reader may undergo as little pains as may be, after he can readily divide any number by a single figure I would advise him to begin at the break in page 63 *and read till he come to the Example and there stop: Then let him begin at the first break in page* 81, *and read to the bottom of page* 90. *After which for about a week I think it expedient that the Learner exercise himself a matter of four hours each day, taking any numbers for Examples to work on that first come to hand. By that time I make no question but the learner will be pretty expert at the work, and by farther practise will grow every day more dextrous and handy at it than other. Then if he pleases he may make tryal of*

the

the first way of Division, and if I mistake not he will easily and speedily get the knack of it. Having thus mastered Division the remaining part of the Book will be but a play to him. If the learner loves his ease and is sparing of his pains, the four ways to ease the Toil of Division to be found at page 93, *to page* 103, *will stand him in great stead. At page* 77 *I would have this scheme inserted. At page* 103 *I would have this added*—

If the dividend and divisor have two, three or more nulla's a piece, lay aside the nulla's and fall to work, the result will

E	36 18	
D	3950	C 1 10

be the same, as if they had not been rejected. As to divide 24000 *by* 600, *I dash out the two nulla's that are together occupying the first and second* [illegible] *of each number, so I have* 240 *and* 6. *Now* 240 *divided by* 6, *the result thereof is* 40, *which is also the quotient of* 24000 *divided by* 600. *These preparatories being premised, I leave the Reader to the perusal of the Treatise and so bid him God speed.*

J

James

~~James~~

Brough

James Brough

Book 1711

Lent to francis Brogs

8 shillings & upon agust

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THE FIRST BOOK OF *ARITHMETICK*: WHOSE SUBJECT MATTER IS WHOLE NUMBERS.

BEfore I come to Numeration, (the first considerable thing in Arithmetick) I must mind the Reader to mark, that in every Number the first figure towards the right hand occupyeth the place of *unites*, the second of *tens*, the third of *hundreds*, the fourth of *thousands* and so forwards; encreasing every time by *ten*. As in the number A 4444, the figure

gure 4 in the firſt place towards the right hand ſignifies four *units*, in the ſecond four *tens* or ten times four *units*, in the third four *hundred*, or a hundred times four *units*, and ſo on. Though in the number A each place be poſſeſſed by the ſame figure (which I made choice of, that the increaſe of each place by ten might be the more eaſily diſcerned) yet if the figures had all been different the ſame thing had happened out, as in the number B 4362, 6 in the ſecond place ſtands for as many *tens* as it ſelf contains *units*, *viz*. 6, and ſo of the reſt. It had alſo been the ſame thing had two or three of the figures been *nulla's*, as in C 4000, where the firſt figure ſignifies the ſame it did in A, *viz*. 4 *thouſand*: For in this caſe no regard is had to the figures, but to the places they poſſeſs in a number. I come now to

NUMERATION.

NUM

The firſt Problem or uneaſie Queſtion.

THe buſineſs of Numeration is to do two things. 1. To read any number

ber put down in Writing or otherwise. 2. Aptly to express by figures any number taken by the Ear, or laid down in Writing or otherwise in words at length.

The first Part.

As to the first of these, for the ease of the Reader, I'll instruct him how to read a lesser number first, and then proceed to a greater.

(1.) Example.

Therefore to read the number 6428, I begin at the left hand and read it thus, *a hundred times sixty four units, twenty eight units, or ten times two units, and eight units.* For if 64 be considered as an intire number having never another figure annexed to it, no doubt but tis rightly expressed by the words *sixty four*, but because with respect to the two following figures the figure 4 occupies the place of hundreds, therefore to the signification they have when alone must in this case be added that of hundreds, and so they signifie *sixty four hundred* or a *hundred times sixty four*, to which add the number expressed by the two last figures, *viz.* 28, which is *twenty eight* units, you have the whole number expressed as before. You may al-

ſo read it thus, *Ten times ſix hundred forty two*, and *eight* units, becauſe the figure 2 poſſeſſes the place of tens. All which explains the way commonly uſed, which is to read the number thus, *ſix thouſand four hundred twenty eight*, adding *thouſand* to the firſt figure on the left hand, *hundred* to the ſecond, and *ten* to the third. By the way I would have the reader take notice that of that infinite number of numbers that may be in nature, there's but few of them that have a proper word whereby one may expreſs them: As above a thouſand no number has a peculiar word to expreſs it but a thouſand times a thouſand, which is uttered by one word, *viz. Million.* Therefore in reading a great number one is forc'd to repeat the ſame word twice or thrice over as you'll ſee by and by.

(2.) *Example.*

I come now to a greater number, *viz.* 39425687243. To read this or any other great number you muſt put pricks under the laſt figure on the right hand and miſſing the two figures 4 and 2 that follow next toward the left hand, under the fourth, *viz.* 7, miſſing two again under the

7th. 5, & so forward toward the left hand as far as the number will permit. Where observe that each member of the number consists of three figures except the first, which may consist of one figure or two as here. The next thing I must do is to consider each member of the number as an intire number of it self, and so I am to read it, adding the denomination to it which belongs to the last figure which has a prick under it.

Now to express the denomination belonging to each figure that has a prick under it, I must repeat the word thousand as many times as those pricks have other pricks after them. So to read the first member considered as an absolute number I say thus, *thirty nine thousand times, thousand times* a *thousand*, repeating the *word* thousand three times, because the prick under its last figure has three pricks following it; for the next member I say, *four hundred twenty five thousand times a thousand*, repeating the *word* thousand but twice; for the next member, I say, *six hundred eighty seven thousand*, making use of thousand but once, the last member is *two hundred forty three*. So that the whole number is read thus, *Thirty nine thousand times, thousand times a thousand* | *Four hundred twenty five*

thousand times a thousand | *six hundred eighty seven thousand* | *two hundred fourty three.* The Reader may remember that before I came to instruct him how to read a great number, I told him that amongst all the numbers above a thousand, there was but one that had a single word whereby it might be expressed, and that was a thousand times a thousand which is expressed by the *word* Million. Now if in reading a great number one would make use of the word Million; He may do it thus,

(3.) *Example.*

As to express this number in words by Millions 4643942568 7243. Divide this number into parts as thou wast taught before. You see therefore according to this dividing the number the seventh figure *viz*, 5. has the place of millions, therefore let that figure have two pricks under it for distinctions sake. Then passing over the next point under 9, put two points also under 6 possessing the seventh place after the figure 5, and so forwards as far as the number will permit, putting two pricks under every seventh place Having now under the two seventh places, (the number

ber admitting no more) clapt two points, you may read the number thus, *Forty ſix millions of millions | four hundred thirty nine thouſand of millions | four hundred twenty five millions | ſix hundred eighty ſeven thouſand | two hundred forty three.*

(4.) *Example.*

Whenſoever a number has ſome *nulla*'s mingled with its ſignificant figures, however the gradual expreſſing of it may be interrupted or broken off, yet the main denomination continues just as if all the figures were ſignificant. As ſuppoſe I was to read this number 39020080003, which is the ſame with the former, but that ſome *nulla*'s are put in the room of ſo many ſignificant figures. Its members or pieces being diſtinguiſhed or laid out by the points (according to the way before taught) thus you may ſpeak it in words, *thirty nine thouſand times, nine thouſand times a thouſand | twenty thouſand times a thouſand | eighty thouſand | and three.* If after the firſt member 39, there had been never a ſignificant figure but all *nulla*'s it had kept its old denomination.

The second Part.

I come now to the 2d. part of Numeration, *viz.* To express by figures any number spoke or wrote in words at length. To do w^h observe that the number thus to be expressed (if proposed to you aright) consists of several members or parts, each of which parts except the first, (which may have one or two figures) has three figures, to the last of which belongs the denomination you add to the simple signification of all the three.

(1.) *Example.*

As suppose this number were proposed to me to be put into figures, *five hundred forty three millions* | *three hundred twenty seven thousand* | *two hundred forty six.* I express the first member by the figures 543 without taking notice of the denomination *i. e.* a thousand times a thousand that appertains to them, for by reason of the two other members that follow, the last figure of the first member under which is the prick must of necessity have that denomination. Therefore I look upon them as a bare number of units. The second mem-

member (*three hundred twenty ſeven thouſand*) I expreſs by 327 juſt as if they were a ſimple number of units, and ſo I add them to the former member. The laſt member I expreſs by 246, which I alſo add to the other two members and the work is done; the number in figures being 543327246.

(2) *Example.*

But ſuppoſe the number was this, *viz. four hundred three millions of millions* | *ſix thouſand* | *two hundred.* To expreſs the firſt member, *viz. four hundred three*, &c. you ſee two ſignificant figures muſt be employed, but that the firſt figure on the left hand may poſſeſs the place of hundreds, there muſt of neceſſity be three figures, and therefore in the place of tens (of which there is here no mention) you muſt put a *nulla*, and thus it ſtands 403, with relation to the two following members; the ſecond member is *ſix thouſand*, without any mention of hundreds or tens, therefore thoſe two places muſt have a *nulla* each, and ſo tis put into figures 006. The laſt member *two hundred* makes no mention of tens or units, ſo that it muſt have two *nulla*'s after the hundred, and ſtands thus 200, all which put together is 403006200.

But if the reader remembers what I told him at the beginning of this Treatiſe concerning the encreaſe of each place by tens, he may eaſily write down any number in figures thus, as the firſt number in page 8, for *ſix* I write down 6 at the right hand, for *forty* or four tens I write down 4, which being in the ſecond place ſignifies four times ten or forty, for two hundred I write down 2, which becauſe it poſſeſſes the third place ſignifies two hundred, for ſeven thouſand I write down 7 towards the left hand, which becauſe 'tis in the fourth place, *i. e.* of thouſands ſtands for ſeven thouſand, again for twenty I write down 2, becauſe in the fifth place, whoſe denomination is tens of thouſands, therefore it ſtands for two times ten thouſand, *viz.* twenty thouſand, for three hundred thouſand, I write down 3 which becauſe it is in the ſixth place ſtands for as much. After the ſame manner for the laſt member put down 543 and the work is done; the number in figures being 543327246.

The end of Numeration

3145678 234 67 2132

ADDI-

Numeration done

Book

ADDITION.

Addition Substraction

The second Problem.

Multyplycation Division

ADdition is the making of two or more numbers one. This compound number is called the ſum of the particular numbers that conſtitute its compoſition.

I ſuppoſe the Reader can already joyn any number under ten to any other number whatſoever, as 3 added to 4 makes 7, 64 added to 8 is 72, and ſo of the reſt. I come now to the principal buſineſs of Addition, which is to find the ſum of many numbers given. To do this I write the ſeveral numbers whoſe ſum I am to find under one another, units under units, tens under tens, hundreds under hundreds, thouſands under thouſands, &c. This being done, I begin at the top of the rank of units (though if *I* began at the bottom 'twere the ſame thing) and add them all together, if their ſum conſiſts but of one figure *I* write it down under the rank of units, if of two *I* write the figure on the right hand under the rank of units, and the left hand figure *I* carry to the rank of tens.

Hav-

Having finiſhed the firſt rank, *I* go to the ſecond, *viz.* that of tens, *I* begin at the top figure, and add them all together (as *I* did before) putting to their ſum what *I* carried from the rank of units, if this ſum conſiſt of but one figure, *I* write it under the rank of tens, if of two I write only the right hand figure under tens, and carry the other to the next place, *viz.* that of hundreds, and ſo for the other ranks that follow towards the left hand. The thing though eaſie in it ſelf will be much eaſier by the following Examples.

(1.) *Example.*

To find the ſum of theſe two numbers, *viz.* 432 and 245, I place them one under another, the 5 that ſignifies five units under 2 that ſignifies two units, 4 that ſignifies 4 tens, under 3 that ſtands for three tens, and 2 that ſtands for two hundred under 4 that ſtands for 4 hundred, the numbers being thus plac'd I draw a line under the lowermoſt number, under which I intend to write the ſum.

$$\begin{array}{r} 432 \\ 245 \\ \hline 677 \end{array}$$

Now I go about my work, beginning at the uppermoſt figure of the firſt rank, to wit, of units, *I* ſay 2 more 5 is 7, and I write 7 under the rank of units, and go-

going to the ſecond rank I ſay 3 more 4 is 7, which I write under the rank of tens. I come at length to the third rank and ſay 4 more 2 is 6, which I place under the rank of hundreds and the work is done, ſo that 677 wrote under the ranks is the number ſought.

(2.) *Example.*

To find the ſum of 459, and 565, firſt I place them as before. 2dly. I ſay 9 more 5 is 14 units, therefore I write 4 under the rank of units, and keep 1 for the next rank. Coming to the ſecond rank, I ſay 1 ten that I kept more 5 tens is 6 tens, which added to 6 is 12 tens, which makes one hundred more 2 tens, therefore *I* write theſe two tens under the rank of tens and keep 1 hundred for the next rank, which is that of hundreds. Being come to the laſt rank, *I* ſay 1 hundred that I kept more 4 is 5 which added to 5 is 10 hundred, which makes one thouſand, which I keep for the fourth rank, *viz.* that of thouſands. And becauſe after 10 hundred or a thouſand, theres never a hundred, *I* put a *nulla* in the third place, which adds nothing; but only it cauſes the figure following to occupy the place of

$$\begin{array}{r} 459 \\ \underline{565} \\ 1024 \end{array}$$

of thouſands. Laſt of all, finding nothing more to be added, I place 1 which *I* kept in the place of thouſands, and ſo *I* know that 1024 is the number ſought.

(3.) *Example.*

To find the ſum of 575 and 425, *I* place them firſt as in the former examples, ſecondly *I* ſay 5 more 5 is ten units, which I reſerve for the next rank, then *I* write a *nulla* under the firſt rank, becauſe never a unit follows the ten units, ſo going to the ſecond rank *I* ſay 1 that *I* reſerved more 7 is 8, 8 more 2 is ten tens or a hundred, which *I* reſerve for the next rank, and *I* write *nulla* under the ſecond rank. Going to the third rank, *I* ſay 1 that *I* reſerved more 5 is 6, 6 more 4 is 10 hundreds or a thouſand, which *I* reſerve for the fourth rank, and *I* write 0 under the third rank. Laſtly having nothing more to add, I clap 1 thouſand that *I* kept in the fourth rank, and ſo *I* know 1000 is the number ſought.

575
425
1000

(4.) *Example.*

To find the ſum of 2000, 3000 and 4000, firſt *I* place them as is uſual, ſecondly

condly *I* ſay (beginning at the firſt rank towards the right hand) 0 more 0 more 0 is 0, which *I* write under the firſt rank. Going to the 2d. *I* ſay, 0 more 0 more 0 is 0, and I write this under the 2d. rank, at the third rank, I ſay 0 more 0 more 0 is 0 which *I* write under this ſame rank. Laſtly coming to the fourth rank I ſay 2 more 3 is 5, 5 more 4 is 9, and I write 9 under that ſame rank, ſo I am aſſured 9000 is the number ſought.

2000
3000
4000
9000

If it ſhould happen the Reader ſhould have many numbers to caſt up into one ſum. As ſuppoſe he was to caſt up the ſum in the margent conſiſting of twelve ſeveral numbers, *I* take the ſum of the four uppermoſt, after the manner juſt before taught, which ſum I write by it ſelf. Then I take the ſumme of the 4 next and write it under the former. *I* do the ſame with the four laſt numbers, and ſo finding their ſum my work is done.

4567	
7919	
3488	
5896	
	21870
7685	
3125	
2635	
8426	
	21871
7934	
2568	
1297	
2354	
	14153
	57894

I ſup-

I suppose one pound to contain twenty shillings, each shilling twelve pence, each peny four farthings.

l.	*s.*	*p.*	*f.*
1	6	2	1
5	7	6	5
8	9	9	6
15	3	8	0

I suppose one yard to contain three foot, one foot twelve inches, one inch three barly-corns.

y.	*f.*	*i.*	*ba. co.*
1	6	2	1
5	7	6	5
8	9	9	6
21	2	9	0

I suppose one barrel to contain four firkins, one firkin nine gallons, one gallon four quarts.

b.	*f.*	*g.*	*q.*
1	6	2	1
5	7	6	5
8	9	9	6
20	0	12	0

This

This Addition differs nothing from the adding of numbers, for beginning with the rank of farthings, I say 1 and 5 is 6 and 6 is 12, now 12 farthings being 3 pence, *I* write 0 under the rank of farthings, and carry 3 pence to the rank of pence, then *I* say 3 *I* carried and 2 is 5 and 6 is 11 and 9 is 20, which is 1 shilling and 8 pence, I write 8 under the rank of pence, and carry 1 shilling to the rank of shillings, then I say 1 carried and 6 is 7 and 7 is 14 and 9 is 23, which is 1 pound 3 shillings, so writing 3 under the rank of shillings, I carry 1 to the rank of pounds, saying 1 I carried and 1 is 2 and 5 is 7 and 8 is 15, which I write under the rank of pounds. The like is to be observed in all other examples.

Proof of Addition.

As to the proof of Addition I shall shew the Reader but one way how to prove it, and that is by casting out all the nines, as well out of the numbers to be cast up, as out of the sum it self. The way to cast the nines out is this, you are to look upon all the figures as if they were of the first rank or digits. In performing the work you need observe no other order but what is necessary for preventing the omission of any figure or passing it over untoucht.

Whilst

Whilst you are about the business, whensoever you make a number greater than 9 by adding the figures, the 9 being cast out joyn the remainder with the next figure, till you have run through all the figures, if any thing remain set it by, if nothing remain set by 0.

Example.

To cast 9 out of these numbers beginning at the uppermost number, I say 4 more 5 is 9 which I cast away, as also the figure 9 which follows, nothing remaining after the nines are cast away I can carry nothing to the first figure on the left hand of the next number.

A	459	
B	565	7
	1024	7

So I say 5 more 6 is 11 out of which 9 being cast there remains 2, which added to 5 is 7, which being under 9 I write it against the number B, but if nothing had remained, after the casting out the 9's, in stead of 7 I would have wrote 0. After the same manner I run over the figures of the sum, and say 9 being cast out of 10 there remains 1 which more 2 is 3, 3 more 4 is 7. The remains being equal I thence conclude the work is rightly done.

SUB-

SUBTRACTION.

The third Problem.

SUbtraction is the finding out the difference between two numbers given, or in other words to find how much the subtrahend is exceeded by the number you are to subtract it from, this excess is called the remainder. The finding out of which is all that Subtraction pretends to teach. I suppose the reader is able to subtract any number under 9 from another under 19, as 3 from 4 and there remains 1, 8 from 14 and there remains 6, &c. It remains that we know the difference of any greater numbers whatsoever, which thing because you cannot do it all at once, it must be done piece-meal in manner following. First I write the less number under the greater, according to the manner observed in Addition. Secondly I begin with the first rank toward the right hand, *viz.* that of units, and subtract the lower figure from that just over it, and write the remainder under that rank, then I go to the next rank and do the like, and so onward till I have run through them all. Thirdly, if the

the lower number chance to be greater than that over it; in this caſe you muſt borrow 1 ten from the next number toward the left hand, which for that reaſon will be leſs by one than it was. The work being thus performed, the figures wrote underneath are the remainder ſought. The examples following will make this extream eaſie.

(1.) *Example.*

869
234
635

To find the difference between theſe two numbers 869 and 234, firſt I place the leſſer under the greater juſt as I did when *I* was about Addition; then I draw a ſtroke under the lower number, 2dly beginning at the firſt rank I ſay 9 leſs 4 is 5, which *I* write under this firſt rank. Going to the ſecond rank I ſay 6 leſs 3 is 3, and I write 3 under this ſame rank. Afterwards I go to the third rank and ſay 8 leſs 2 is 6, and I write 6 under the laſt rank. The work being done *I* know the difference ſought is 635.

(2.) *Example.*

To find the difference between 678 and 489, I place them as in the firſt example

9 units

9 units from 8 I cannot, therefore I borrow one ten from the 7 tens in the next rank, and write 6 in the room of it, ſo that 8 and 10 borrowed is 18, therefore *I* ſay 18 leſs 9 is 9, which I write under the firſt rank, then going to the ſecond rank *I* ſay 8 from 6 I can't, therefore I borrow 1 ten from the next rank, wherefore in ſtead of 6 I write 5, now 6 added to one ten borrowed is 16, therefore 16 leſs 8 is 8, which I write under the ſecond rank. Laſtly coming to the third rank, I ſay 5 leſs 4 is 1 which I write under the third rank, ſo I know that 189 is the difference ſought.

56
678
489
189

(3.) *Example.*

To find the difference between 842 and 405, firſt I place them according to the accuſtomed manner, ſecondly beginning at the firſt rank I ſay 5 from 2 *I* cannot, therefore *I* borrow one ten from 4 and write 3 in ſtead of it, ſo that 1 ten borrowed more 2 is 12, therefore *I* ſay 12 leſs 5 is 7 which *I* write under the firſt rank. Going to the ſecond I ſay 3 leſs 3 is 0, which 3 *I* write under the ſecond rank. At the laſt rank *I* ſay 8 leſs 4 is 4, *I* write 4 under this ſame rank, and ſo *I* know 437 is the difference ſought.

3
842
405
437

(4.) *Exam-*

(4.) *Example.*

To find the difference between 346 and 246, first *I* rank them according to the accustomed manner, secondly beginning at the first rank I say 6 less 6 is o, and I write o under this rank. Going to the second rank, *I* say 4 less 4 is o, which *I* also write under this rank. At the last rank *I* say 3 less 2 is 1 and *I* write 1 under this same rank, so I know that 100 is the difference sought.

346
246
100

(5.) *Example.*

To find the difference between 800 and 200, first I place them as usual, secondly at the first rank I say o less o is o, and I write o under this rank, going to the second I say also o less o is o and I write o under this rank too. At the third I say 2 from 8 is 6, and I write 6 under this rank, and so I know 600 is the difference sought.

800
200
600

(6.) *Example.*

To find the difference between 900 and 432, first I rank them as is usual, secondly at the first rank I say 2 from o I cannot, therefore I borrow 1 ten from the o following, and over it I write less 1 having taking 1 from it,

-8-1
900
432
498

it, the 1 ten borrowed and 0 is 10, so I say 2 from 10 and there remains 8 which I write under the first rank. Going to the second I say 3 from less 1 I cannot, therefore I borrow 1 ten from the 9 and write 8 in stead of it. Now the 10 borrowed added to less 1 is 9, I say therefore 3 from 9 and there remains 6, and I write 6 under this rank. Lastly at the third rank, I say 8 less 4 is 4 and I write 4 under this same rank, so that I know 468 is the difference sought.

I shall add two Examples more leaving them to the Readers consideration.

—1—1—1	0—1
1 2 0 0 0	11234
6 0 7 2	10725
5 9 2 8	00509

By —1 I mean that the number under it is less by 1.

Some in stead of lessening the figure from whence they borrow 1 ten by a unite, like rather to encrease the figure under it by 1. As to work this following example.

(7.) *Example.*

I say 2 from 0 I cannot, therefore I borrow 1 ten from the 0 that occupies the place

place of tens by adding 1 to the 3 right under it, ſo I ſay 10 I borrowed leſs 2 is 8, and I write 8 under the rank of units. Going to the ſecond rank I ſay 4 from 0 I

```
 900
 432
  11
----
 468
```

cannot, therefore I borrow 1 ten from the figure 9 which is in the third rank by adding 1 to 4 wrote under 9, then ſay I at the ſecond rank 10 borrowed leſs 4 is 6, and I write 6 under this rank, Laſtly I ſay at the third rank 9 leſs 5 is 4, I write 4 under this rank, and I know 468 is the difference ſought.

This Rule may be performed by beginning at the left hand and going on towards the right, if one remember to keep 1 ten of each rank for the next, when you ſee its under figure is greater than that over it, or when both its figures being equal, the under figure of the next rank exceeds that over it. One example will make this plain.

(8.) *Example.*

To ſubtract 77989675 from 257389560 I place them as I did before, and beginning

```
257389560
 77989675
---------
179399885
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at the ninth and laſt rank, I ſay 2 leſs 0 is 2, but becauſe in the eighth rank 7 exceeds 5, therefore I write only 1 under the ninth rank, and I keep

keep 1 which when added to the eighth rank is 10, and coming to the eighth I ſay 10 reſerv'd more 5 is 15, which leſs 7 is 8; but becauſe the figures of the 7th. rank are equal, *viz.* 7 and 7, and at the 6th. rank 9 exceeds 3 under which it is placed, I write only 7 under the eighth rank, and I carry 1 ten to the 7th. then coming to the 7th. rank I ſay 7 more 10 that I carryed is 17, which leſs 7 is 10, but I write only 9 under this rank; coming to the 6th. I ſay 10 carried more 3 is 13 which leſs 9 is 4, but becauſe 8 and 8 in the 5th. rank, and alſo 9 and 9 in the 4th. rank are equal in each rank, and at the 3d. rank 6 exceeds 5 under which 'tis placed, I write only 3 under the 6th. rank, and 9 under the 5th. and 9 alſo under the 4th. Then coming to the 3d. I ſay 10 more 5 is 15, which leſs 6 is 9, but I write but 8, and at the 2d. rank, I ſay 10 more 6 is 16 which leſs 7 is 9, but I write 8 only: Laſtly at the firſt rank I ſay 10 more 0 is 10, which leſs 5 is 5, and I write 5 under this rank, ſo I know 17939985 is the difference ſought.

---

place of tens by adding 1 to the 3 right under it, so I say 10 I borrowed less 2 is 8, and I write 8 under the rank of units. Going to the second rank I say 4 from 0 I cannot, therefore I borrow 1 ten from the figure 9 which is in the third rank by adding 1 to 4 wrote under 9, then say I at the second rank 10 borrowed less 4 is 6, and I write 6 under this rank, Lastly I say at the third rank 9 less 5 is 4, I write 4 under this rank, and I know 468 is the difference sought.

900
432
11
468

This Rule may be performed by beginning at the left hand and going on towards the right, if one remember to keep 1 ten of each rank for the next, when you see its under figure is greater than that over it, or when both its figures being equal, the under figure of the next rank exceeds that over it. One example will make this plain.

(8.) *Example.*

To subtract 77989675 from 257389560 I place them as I did before, and beginning at the ninth and last rank, I say 2 less 0 is 2, but because in the eighth rank 7 exceeds 5, therefore I write only 1 under the ninth rank, and I keep

257389560
77989675
179399885

keep 1 which when added to the eighth rank is 10, and coming to the eighth I ſay 10 reſerv'd more 5 is 15, which leſs 7 is 8, but becauſe the figures of the 7th. rank are equal, *viz.* 7 and 7, and at the 6th. rank 9 exceeds 3 under which it is placed, I write only 7 under the eighth rank, and I carry 1 ten to the 7th. then coming to the 7th. rank I ſay 7 more 10 that I carryed is 17, which leſs 7 is 10, but I write only 9 under this rank; coming to the 6th. I ſay 10 carried more 3 is 13 which leſs 9 is 4, but becauſe 8 and 8 in the 5th. rank, and alſo 9 and 9 in the 4th. rank are equal in each rank, and at the 3d. rank 6 exceeds 5 under which 'tis placed, I write only 3 under the 6th. rank, and 9 under the 5th. and 9 alſo under the 4th. Then coming to the 3d. I ſay 10 more 5 is 15, which leſs 6 is 9, but I write but 8, and at the 2d. rank, I ſay 10 more 6 is 16 which leſs 7 is 9, but I write 8 only: Laſtly at the firſt rank I ſay 10 more 0 is 10, which leſs 5 is 5, and I write 5 under this rank, ſo I know 17939985 is the difference ſought.

(9.) *Example.*

l.	*s.*	*p.*	*f.*
8	7	8	5
3	4	9	6
5	2	10	3

(10.) *Example.*

y.	*f.*	*i.*	*b.-c.*
6	8	7	4
3	9	3	5
2	2	3	2

(11.) *Example.*

b.	*f.*	*g.*	*q.*
5	7	4	6
2	8	3	5
2	3	1	1

The Proof of Substraction.

To prove whether substraction be rightly perform'd do only this, cast the 9's out of

of the greater number, and alſo out of the ſubtrahend and remainder as if one number; (the way how to caſt out 9 was ſhewed in Addition) if the ſame number remain, the work is rightly performed, if not, the work muſt be repeated.

As for Example.

To prove whether 876 be the true remainder, 1469 being ſubſtracted from 2345, I ſay 2 more 3 is 5, which added to 4 is 9 which I caſt away and there remains 5. Then I ſay 1 more 4 is 5, which added to 6 is 11, 2 above 9, I fling away 9 and carry 2 to the next, *viz.* 9, which added together make 11, 2 above 9, which I fling away, and add 2 to 8 the firſt figure of the remainder which makes 10, 1 above 9, which I caſt away and add 1 to 7 which makes 8 which added to 6 is 14, 5 above 9 which being caſt away there remains 5, the ſelf-ſame figure that remain'd when all the 9's were caſt out of the uppermoſt figure, therefore I am ſure the work is rightly performed.

2345	[5
1469	
876	[5

MULTIPLICATION.

Multiplication or multiplying one number by another, is a finding out a number that contains the number multiplyed as often as the multiplier contains unity. Touching the business of Multiplication two numbers are necessary to the performing of it, of which one is called the Multiplicand or Multiplied, the other the Multiplier. The first is a number compounded of it self repeated. The second is a number which by its unities shews how often the former ought to be repeated.

As suppose the numbers A and B were to be multiplied, of which let A be compounded of it self so many times repeated as there are unities in B, and by that means let the number C be made; the number A will be the multiplyed, B the multiplier, and the number produced by the Multiplication C. But which of them is to be call'd the Multiplicand, and which the Multiplyer depends upon the manner you observe in your work, For if the number

A	25
B	4
C	100

A be

A be compounded of it ſelf ſo many times repeated as there are unites in B, and one ſhould ask what number is four times 25 units, the Multiplicand then is A, and the Multiplier B. Again if 'twere ask'd how many is twenty five times four units, in that caſe B is the Multiplicand and A the Multiplier. However which ſoever of them is Multiplicand or multiplier being multiplyed one into another the ſame number C is produced ; As is evident from the *16th. Prop. of the 7th. Book of the Elements of Geometry.*

There is another Definition of Multiplication which I think it neceſſary to acquaint the Reader with, and 'tis this, multiplication is the finding out a number that has the ſame ration (or relation with reſpect to greatneſs and littleneſs) to the multiplicand, the Multiplier has to unity, for if in the preceding Example, the number C produced by the multiplying B into A contains A, as often as B contains unity, the number C will become as manifold of A as B of unity. Upon which account (according to the 20th. *Definition of the 7th. Book of the Elements of Geometry*) The product has the ſame ration to A. that B has to unity, ſo that the four numbers, *viz.* C the number produced, A

the multiplicand, B the Multiplier, and unity become four terms in a Geometrical Proportion. If this ſhould ſeem a little dark to thee, Reader, prethee ſtay till thou comeſt to the Rule of Three, for there I intend to diſcourſe at large of Proportions (it being a thing of ſo great weight in Arithmetick,) from whence thou mayſt fetch light enough to diſpel the darkneſs in an inſtant which ſeems to hang over this place. As in Addition and Subſtraction the ſum total and difference between two numbers was not found all at once, but at ſeveral times, ſo in Multiplication (which is nought elſe but a *compendious way of Adding*) each figure of the multiplier muſt be multiplied into the figures of the Multiplicand right over it, after which all the Partial or ſingle products being added together; the whole product comes to be known. Therefore the buſineſs of Multiplication depends very much upon the Multiplying one digit or ſingle number into another, which ought to be learnt perfectly before we undertake the multiplying of numbers conſiſting of divers figures.

The Fourth Problem.

How any two numbers being given to find their product.

Its

Its first part,

To find the product of any two digits.

The first way of doing it.

The first (and really 'tis both a pleasant and an easie one) is by a Table said to be found out by *Pythagoras.* Touching this Table I shall shew the Reader first how to make it. Secondly, The Use thereof.

The Multiplication Table.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

The framing of the Table.

As to the first, range the nine Cyphers in two ranks, each rank beginning with 1, and one of them running across from the

left hand to the right, the other going down right from top to bottom. Then from each figure begin the other ranks as well the croſs ones as thoſe downright, by adding continually each figure to it ſelf. As for inſtance to make the ſecond rank both croſs and downright, which begins with 2, add 2 to it ſelf, and ſo 4 is made, which is to occupy the ſecond place in each rank, to 4 add again 2 that it may be 6, to 6 add 2, that it may make 8, to 8 add 2, that it may be 10 and ſo on, placing each of them in their places which belongs to them in both ranks till you come to the laſt, after the ſame rate you may make the other ranks. As if to 8 you add 8 it becomes 16, to which 8 again being added is 24 and ſo forward. By this means you have the two ranks croſs and downright, beginning each of them at 8. This is the way of framing the Table, which to multiply any ſingle figure into another ſingle one, uſe after this manner.

The Uſe of the Table.

As ſuppoſe a man would know what number is produced by multiplying 7 by 8, ſeek either of them, as ſuppoſe 7 in the croſs rank and the other, *viz.* 8 in the

down-

downright rank, pitch upon the number that ſtands in both ranks, or that ſtands at the place where the two ranks cut one another, and that is the number you deſire as here 56. The ſame way you may find out the product of any other two numbers multiplyed one into another.

The ſecond Way.

But becauſe this multiplication Table may be ſometimes not at hand, I'll ſhew the Reader how to multiply one digit by another, after another manner.

(1.) *Example.*

As ſuppoſe I was to multiply 6 by 8, I'll do it preſently thus; on the left hand of the croſs I write the two numbers to be multiplyed one under another. On the right hand of the croſs I write their differences from 10 over againſt them, as 4 againſt 6, and 2 againſt 8, that done multiply the differences and write their product, if it conſiſt but of one figure, directly under them as 8 here. Laſtly ſubſtract croſwiſe either 2 from 6 or 4 from 8 & write the difference juſt under

6	X	4
8		2
4		8

left hand to the right, the other going down right from top to bottom. Then from each figure begin the other ranks as well the croſs ones as thoſe downright, by adding continually each figure to it ſelf. As for inſtance to make the ſecond rank both croſs and downright, which begins with 2, add 2 to it ſelf, and ſo 4 is made, which is to occupy the ſecond place in each rank, to 4 add again 2 that it may be 6, to 6 add 2, that it may make 8, to 8 add 2, that it may be 10 and ſo on, placing each of them in their places which belongs to them in both ranks till you come to the laſt, after the ſame rate you may make the other ranks. As if to 8 you add 8 it becomes 16, to which 8 again being added is 24 and ſo forward. By this means you have the two ranks croſs and downright, beginning each of them at 8. This is the way of framing the Table, which to multiply any ſingle figure into another ſingle one, uſe after this manner.

The Uſe of the Table.

As ſuppoſe a man would know what number is produced by multiplying 7 by 8, ſeek either of them, as ſuppoſe 7 in the croſs rank and the other, *viz.* 8 in the down-

downright rank, pitch upon the number that ſtands in both ranks, or that ſtands at the place where the two ranks cut one another, and that is the number you deſire as here 56. The ſame way you may find out the product of any other two numbers multiplyed one into another.

The ſecond Way.

But becauſe this multiplication Table may be ſometimes not at hand, I'll ſhew the Reader how to multiply one digit by another, after another manner.

(1.) *Example.*

As ſuppoſe I was to multiply 6 by 8, I'll do it preſently thus; on the left hand of the croſs I write the two numbers to be multiplyed one under another. On the right hand of the croſs I write their differences from 10 over againſt them, as 4 againſt 6, and 2 againſt 8, that done multiply the differences and write their product, if it conſiſt but of one figure, directly under them as 8 here. Laſtly ſubſtract croſwiſe either 2 from 6 or 4 from 8 & write the difference juſt under

6	X	4
8		2
4		8

the numbers to be multiplied as you ſee in the ſcheme and the work is done. But when the number produced by multiplying the 2 differences from 10 conſiſts of two figures, write the laſt under the differences from 10, the other keep in your mind to be added to the remainder that is to be written under the numbers to be multiplied.

(2.) *Example.*

To Multiply 5 by 7, I write 5 and 7 on the left hand of the croſs and their differences from 10 on the right hand over against them, as I did in the former Example; then I multiply theſe differences, *viz.* 3 into 5, and becauſe the product 15 conſiſts of two figures, I write

5	X	5
7		3
3		5

only the laſt, *viz.* 5 under the differences from 10, and keep the other in my mind. This done I ſubſtract croſswiſe either 5 from 7 or 3 from 5, and to the remainder 2 I add 1 the number I kept in mind which makes 3, which number I write under the numbers to be multiplyed, as you ſee in the ſcheme and the work is done.

The

The second part of the fourth Problem; How to multiply a number consisting of many figures into another that consists of many figures also.

Having instructed the Reader how to multiply one single number by another, I come in the next place to shew him how to multiply two numbers consisting of many figures. Therefore to multiply any two such numbers do thus, write down the greater number of the two first for the Multiplicand, and under that write the lesser number for the Multiplier, so that the first figure of the Multiplier may stand just under the first figure of the Multiplicand, the second under the second, and so of the rest. This being done begin with the first figure of the Multiplier towards the right hand, and Multiply it into each figure of the Multiplicand one after another, multiplying the first figure towards the right hand first, and then the rest in their order, after the same manner multiply the multiplicand by the second figure of the multiplier, and after that by the third and so on. But the numbers produced by multiplying each figure of the multiplier into the multiplicand are to be wrote one under another severally, the first figure of each

each product towards the right hand being wrote exactly under the figure of the multiplier that help'd to produce it; theſe partial or ſingle products being reduced into one ſum, make the product ſought. A few Examples will make this mighty plain.

(1.) *Example.*

To find the product of theſe two numbers 24 and 3, firſt I place the leſs number under the greater. Secondly I ſay 3 times 4 is 12, I write the laſt figure under the number 3, and keep one ten for the ſecond place, then I ſay 3 times 2 is 6 and one kept in mind is 7, which I write exactly under the ſecond figure of the multiplicand, and ſo I know 72 is the product ſought.

$$\begin{array}{r} 24 \\ 3 \\ \hline 72 \end{array}$$

(2.) *Example.*

To find the product of 84 by 26, firſt I place 26 under 84; ſecondly I multiply 84 firſt by 6, ſaying 6 times 4 is 24, I write 4 under the 6, *to wit*, the laſt figure of the multiplier, and keep 2 in my mind for the next place, *viz.* that of tens, then I ſay 6 times 8 is 48, and 2 reſerved is 50, ſo I write 0 under the rank of tens

$$\begin{array}{r} 84 \\ 26 \\ \hline 504 \\ 168 \\ \hline 2184 \end{array}$$

tens and place 5 in that of hundreds. Having found the product of 84 by the firſt figure of the multiplier and wrote it down in a line by it ſelf, I multiply the ſecond figure of the multiplier into the multiplicand thus, 2 times 4 is 8 which I write juſt under 2 the ſecond figure of the multiplier, again I ſay 2 times 8 is 16, which I write down immediately after the 8, ſo that I have two partial products which being added together make the product ſought 2184.

(3.) *Example.*

To find the product of 670 by 305, firſt I place 305 under 670, ſecondly I multiply 670 by 5 the firſt figure of the multiplier, ſaying 5 times 0 is 0, which I write under 5 that poſſeſſes the place of units in the multiplier, then I multiply 5 into 7 the ſecond figure of the multiplicand, ſaying 5 times 7 is 35, I write 5 in the ſecond place after the 0 and keep 3 for the next, again I ſay 5 times 6 is 30 to which 3 kept in mind being added makes 33, I write down the 3 on the right hand in the third place after 5 and the other in the 4th. place. So I have

670
305
3350
20100
204350

have the firſt partial product. Then I multiply 670 by 0 the 2d. figure of the multiplier, ſaying 670 0 is 0, ſo I write 0 in the ſecond place which is to be the firſt figure towards the right hand of a ſecond partial product, to be wrote in a line by it ſelf under the former. Laſtly I multiply 670 by 3 the laſt figure of the multiplier, ſaying 3 times 0 is 0, and I write 0 under the third rank after 0 plac'd in the ſecond, again I ſay 3 times 7 is 21, I write 1 in the 4th. rank and keep 2 for the fifth, again 3 times 6 is 18, to which 2 that I reſerved being added makes 20, ſo I write 0 in the fifth rank, and clap the 2 in the ſixth, theſe two partial ſums added together give me the product ſought, *viz.* 204350. Now to prove whether the work be rightly performed I uſe the ſame courſe I took in Addition and Subſtraction, *viz.* caſting away 9. As for inſtance to prove whether the laſt example be truly wrought. Firſt I

	5	
4	X	8
	5	

fling away 9 out of the Multiplicand and Multiplier, after the method uſed for the proof of Addition, and the two remainders, *viz.* 4 and 8 I write by themſelves as in the ſcheme. Afterwards I multiply them one into another, ſaying 4 times 8 is 32, out of which

I caſt

I caſt 9 and there remains 5, which I write alſo by it ſelf. Then I caſt 9 out of the product, and becauſe the ſame figure remains, I know the work is rightly performed.

670
305
3350
20100
204350

But ſuppoſe after tryal the remainders ſhould chance to be unequal the 9 being caſt away, how ſhould one find where the fault was committed? To find out where the fault was committed I'd have the Reader do after this manner, as in this example 3584 being multiplied by 672 the product according to the Rules of Multiplication, is 2408448. Now ſuppoſe there be a fault committed in the working of it I take this courſe to find it out, I caſt 9 out of the number A, and I write the remaining figure on the right hand of it, but drawing a line firſt for diſtinction; then I begin my multiplication after the uſual manner, but ſo that 2 the firſt figure of B the multiplier be firſt of all multiplied into 2 the remaining figure after 9 was caſt out of A the

A	3584	2
B	672	
C	7168	4
D	25088	5
E	21504	3
F	2408448	3

the multiplicand and I write 4 the product directly under it drawing a line first between them. But if so be the number thus produced should consist of two figures fling away 9 and write the remainder. This done I multiply the number A by 2 the last figure of the multiplier, the number produced is C. Now if out of the number C you cast 9 doubtless if the work be rightly performed the same number will remain, *viz.* 4 For this way of tryal is the very same with that we used just now; since two numbers are multiplied into each other A 3584 and 2; and the number by that means produced is C. Now if one would prove this single operation, it may be done after the manner just now taught, *viz.* casting 9 out of the multiplicand A and writing the remaining figure 2 by it self, 9 also being cast out of the multiplier 2, which casting away of 9 can never alter it, unless it should be 9 and so to be changed into 0. If therefore the two separated numbers 2 and 2 one of the multiplicand A, the other of the multiplier 2 be multiplied together so that the product be 4,(out of which had it exceeded 9 you must have cast 9 and reserved the remaining figure) the same figure 4 ought to remain if 9 be cast out of

C.

C. A man being thus ſure that he has done this ſingle operation right, which one might eaſily mend if the caſting away 9 ſhould prove it to be falſe by the inequality of the numbers remaining. After the ſame manner one may be aſſured that the work is duly performed in the other two numbers D and E. As the figure 7 which multiplying the number A produced D, if it multiply the remaining figure after 9 is caſt out of the number A *viz.* 2 that is ſet aloof of, ſo that you may have the product 14, out of which caſt out 9 that you may have the remaining figure 5. The ſelf-ſame figure will certainly remain 9 being caſt out of the number D, if the multiplying the number A by 7 which produced it be rightly performed. Do the like with the reſt.

Before I leave multiplication I'll ſhew the Reader how in ſome caſes he may both ſhorten and make eaſie the work of Multiplication.

The Firſt Caſe.

When either the multiplicand or multiplier, or both, have nulla's at the right hand, let alone the nulla's and multiply the ſignificant figures, to the product add the

the nulla's whether they belong to one or both numbers. For Example the two numbers A and B: multiply 24 without its nulla's by 6, to the product 144 add the two nulla's belonging to A that the whole product may be 14400. Again suppose the numbers were D and E each of them having nulla's at the left hand, setting aside their nulla's I multiply 23 by 5, to the product 115, I add 3 nulla's, one of which belonged to D, the other two to E, that it may be the whole number F produced by the multiplication of D by E.

A 2400	D 230
B 6	E 500
C 14400	F 115000

Hence follows a ready way of multiplying by a number whose first figure is 1 the rest nulla's, as to multiply 24 by 10, 100, &c. your work will be done by putting the nulla's of the multiplier to the multiplicand, that the product may be 240, 2400. The reason is because 1 in multiplication does not at all alter the multiplicand.

The second Case.

Two numbers being given to be multiplied, divide either of them into two or more parts by every of which parts multiply

ply the other number undivided, after which theſe ſingle products being all added together their ſum is the product ſought. As if a man were to multiply 24 by 36, I divide 36 into two parts one whereof is 30 the other 6, both of w^ch I multiply by 24 the ſingle products 720 and 144 being added make the number ſought 864. The ſame number would be found if 36 were divided into 18 and 18, for 24 multiplied by each of them produces 432, which put twice and ſo added or multiplyed by 2 makes the ſame number as before 864.

36	
30	6
24	24
720	144

After the ſame manner if 24 were divided into 3 parts, *viz.* 10, 10 and 4; and 36 were multiplyed by every of them, the ſingle products 360, 360, 144, added together make 864, the ſame that would be produc'd by multiplying 36 by 24 after the uſual manner.

The Reader may make uſe of the ſame brief way after this manner: To one of the two numbers to be multiplied add another number which may ſo increaſe it as to make its laſt figure be a 0: make the other number the Multiplicand and the number thus encreaſed the Multiplier. From the pro-

product substract the number produc'd by multiplying the former multiplicand by the number added, the remaining number is the product that would be made were the two numbers multiplyed after the common way. As suppose 45 were to be multiplyed by 27, to 27 add 3 that it may become 30, then multiply 45 by 30, the product is 1350, from which the number 135 (which is produced by multiplying 45 by 3 the number added) being substracted the remainder is 1215, the same which arises by multiplying 45 by 27. This way will sometimes render the multiplying one single number by another very easie, whether one of the numbers be divided, augmented or diminished, especially since such multiplications are often to be perform'd by the mind only. As if 9 were to be multiplyed by 8, fancy 1 to be added to 9 that it may be 10, then multiply 10 by 8, the product is 80, from which 8 being substracted because of the 1 added to 9, the remainder is the product sought.

The Third Case.

If of the two numbers to be multiplyed one of them may be cut into equal parts, multiply the other by one of those parts, and

and the product by a number which declares what part it is of the whole, the number thus produced is the product sought. As suppose one was to multiply 56 by 20, because 20 can be divided into two equal parts, *viz.* 10 and 10, you must multiply 56 by 10 and the product 560 by 2 (the number that shews what part 10 is of 20 as here a half) the number thus produced 1120, is the same as if 56 were multiplyed by 20. Again suppose 20 were to be divided into other parts as into five, every one of which is 4, multiply 56 by 4 and the product 224 by 5 (that tells 4 is a fifth part of 20) the same number 1120 is produced as before. Furthermore it may be here observed that one of the numbers to be multiplyed may so be divided into equal parts, that the parts themselves may again be divided into other equal parts. For instance, the number 24 that is to multiply 15 may be divided into 4 equal parts, each of which will be 6, which if it multiply 15 the product will be 90, which again multiplyed by 4 (that declares what part 6 is of 24) produces the self same number 360 that 15 multiplyed by 24 does. But because I am to multiply 90 by 4, why may I not divide one of them, *viz.* 4 again into two equal

equal parts? one of which may multiply 90, that it may beget 180, which product again I multiply by 2 (which ſhews what part 2 is of 4) that ſo I may light upon 360, the ſame that 15 multiplyed by 24 produces. Therefore theſe three numbers 2, 2, 90, multiplyed one into another (tis no matter with which two of them you begin your multiplication) will always produce 360, which number muſt be produced by 24 and 15, undivided and unaltered if multiplyed one into another.

I come in the laſt place to ſhew the Reader a brief and eaſie way of multiplying eſpecially when the numbers to be multiplyed conſiſt of many figures each. But before I come to it, I cannot forbear to tell the Reader that this way is in the opinion of divers excellent Arithmeticians the moſt ſafe way that can well, nay poſſibly, be found out to multiply by, for I cannot ſee which way he that pleaſes to make uſe of it can miſtake, be the numbers to be multiplyed never ſo great. Add to this that this very ſleight I am going to ſhew thee will free Diviſion from all its difficulty (which was always accounted very great) ſo that it may very well contend with Addition for eaſineſs, as upon tryal

tryal will be found. The way is this. Make a Table of the multiplicand just as the multiplication Table was made.

Example.

A	3584
B	672
C	7168
D	25088
E	21504
F	2408448

	A
1	3584
2	7168
3	10752
4	14336
5	17920
6	21504
7	25088
8	28672
9	32256

The two numbers A and B are to be multiplyed one into another. Take one of them and make its Table. Take therefore the nine figures and place them at the left hand as in the Scheme. Over against 1 write A the multiplicand, then add it to its self or multiply it by two, write the product over-against 2. To this add A that it may be the number to be placed against 3. To this add A and thou wilt have the fourth to be placed against 4. So go on by adding A till thou hast the nine numbers to be wrote against the nine fi-

figures. By this means you have the Table, which you are to make uſe of in multiplying the number A by any other whatſoever after the ſame manner you uſed *Pythagoras* Table to multiply one digit by another. For every number of this Table contains the number A as often as the figure over againſt it contains unity, as is evident from the frame and contexture of the Table. This being done, let the number A be to be multiplied by B. The numbers A & B being placed one under another was before taught, you are to multiply 2 the laſt figure of B into the multiplicand A; go to the Table and from thence take the number right againſt 2, writing it at C, writing its laſt figure exactly under 2 as was taught before in multiplication. Then you are to multiply 7 the laſt figure ſave one of B into the whole multiplicand A. You have this multiplication already done in the Table over againſt 7. Write the number D taken from thence under, as in the ſcheme, do after the ſame manner with the figure 6, by which you have E. Theſe partial products being added into one ſum, you have the ſelf-ſame number that is produced by multiplying A by B after the common way and that with the greateſt eaſe that can be. I know there are

are other ways of multiplication, as to multiply by *Nepiers Bones*, and by the *Trigonometrical Canon*, but I pass them over, as not proper to be handled here and because those already laid down are sufficient.

The Use of Multiplication.

If one would at any time reduce any thing of a greater denomination to another thing of a less, you need only multiply the greater by the number, denoting how often the greater contains the less and the work's done.

As to reduce 5 shillings into pence, multiply 5 by 12, the number denoting how often a shilling contains a peny, and the product is 60 pence. So to reduce 6 yards into feet, multiply 6 by 3 (because a yard is 3 foot) the product is 18 feet, and so of the like. I come now to division which is thus defined.

Of DIVISION.

DIvision or dividing one number by another, is the finding out a number that contains unity as often as the number divided contains that that divides it. What Division is may also be thus explained. 'Tis the finding out a number that bears the same proportion to unity, that the number divided does to that that divides it, which though in other words differs little or nothing from the former. From whence Reader thou plainly seest what the business of Division is, or what it helps and aids thee to do; to wit, any two numbers being given or known a greater and a less; by dividing the greater by the lesser I find a third which is called a quotient, because it tells how oft the lesser number is contained in the greater. Hereafter I shall call the greater of the two that are known the Dividend, the lesser the Divisor, and the number sought or unknown the Quotient. Therefore to divide one number by another do thus. First place the divisor under the ranks

ranks of the dividend that begin at the left hand, ſeeing that the diviſor do never exceed thoſe figures of the dividend under which 'tis placed. Secondly ſeek by piece-meal (beginning at the left hand) how often the diviſor is contained in thoſe ranks, and writing the quotient found in a half moon by it ſelf, ſubtract the product of the diviſor by the quotient from the ranks divided. Thirdly, Move each figure of the diviſor forward toward the right hand one ſtep, then try by piece-meal how often the diviſor is contained in the figures under which it is placed, and writing the quotient with the former on the right ſide of it, ſubſtract the product of the diviſor by the new quotient from the figures ſtanding over the diviſor. Repeat the ſame work till the diviſor has run through all the figures of the dividend. After all is done the number in the half moon is the quotient ſought. The Examples will make the Rule plain. But becauſe ſometimes the diviſor happens to conſiſt of many figures, and then the work of diviſion is ſomewhat more difficult, I ſhall firſt inſtruct the Reader how to divide by a digit or ſingle number, which being underſtood the other will become more eaſie.

(1.) *Example.*

To find the quotient of 64 divided by 2, first I place 2 under the figure 6, secondly I say 2 is three times in 6, I write 3 in a half moon by it self, and I substract the product of the quotient 3 by 2 from 6 the figure or rank divided, saying three times 2 is 6, 6 from 6 and there remains 0, so I dash out 6 and 2 wrote under it. Thirdly I remove the divisor 2 from the second rank or place possessed by 6 to the first occupied by 4 writing 2 just under it. Then I say 2 is twice in 4, I write 2 with the other quotient on the right hand of it, and I substract from the rank divided which is 4, the product of the divisor 2 by the new quotient which is also 2, saying 2 times 2 is 4, 4 from 4 and there remains 0, that is, there remains nought. Therefore I dash out 4, and 2 that is under it. So the work being finished I know that 32 wrote in the half moon is the quotient sought.

$$\begin{array}{l} \cancel{6}\cancel{4}\ (32 \\ \cancel{2}\cancel{2} \end{array}$$

(2.) *Example.*

To find the quotient of 84 divided by 42. First I place 42 under 84. Secondly,

ly I ſay 4 is twice in 8, I write 2 in the half moon, and I ſubſtract from the rank 8 the product of the quotient 2 by 4 wrote under 8, ſaying 2 times 4 is 8, 8 from 8 and there remains 0, and I daſh out 8 and 4 that is under it. I ſubſtract alſo from 4 occupying the firſt rank of the dividend the product of the quotient 2 by 2 wrote under 4, ſaying 2 times 2 is 4, 4 from 4 and there remains 0. So daſhing out 4 and 2, I know 2 is the quotient.

84 (22	~~8~~~~4~~ (2
~~4~~2	~~4~~~~2~~

The Reader is to take notice that if the diviſor be under a figure of the dividend and there be on the left ſide of it one or two more, thoſe two or three figures are ſuppoſed to ſtand over the diviſor. For example, $\begin{matrix}2\,4\\ \ \ 4\end{matrix}$ you muſt underſtand 24 to be over 4, and 4 to be under 24. Likewiſe in $\begin{matrix}2\,4\,0\,6\\ \ \ \ \ 9\ \ \end{matrix}$ you muſt conceive 240 to be over 9 and 9 under 240, but by no means think 9 is under 6 or 6 over 9. This will be of great conſequence hereafter.

(3.) *Example.*

To divide 6832 by 7. Now becauſe 7 is greater than 6, I muſt not place my diviſor exactly under 6 but under the next figure to it toward the left hand; becauſe the figures of the dividend to be divided

must always exceed the divisor. Secondly I am to ask how often 7 is in 68. Now the Learner will not be able readily to answer this question, therefore I'll shew him how by the help of the multiplication Table he may easily do it after this manner. Let him seek the divisor 7 in either of the ranks, downright or cross rank; then go from 7 in its rank till you meet with 68, or the number nearest to it that is less than it, as here you wont meet with 68, therefore make use of 63, less than it, but the next to it. From the number 63 found go upwards in the rank of 63, if you made use of the cross rank of 7; or a-cross if you made use of the downright rank till you come to the uppermost rank of AB or AC, for the figure placed in that (as here 9) shews you how often 7 is contained in 63, and so in 68 the next less number than it found in the Tables, which number 9 I write in the half moon, then I substract the product 63 of the divisor 7 by the quotient 9 from 68 plac'd over the divisor, the remainder 5 I write over 8. Then dash out 68 and 7 as useless. This operation being done, move the divisor one step forward toward the right hand, and write it exactly

 5
~~6~~832 (9
 ~~7~~

ly

ly under the following figure of the Dividend.

As here write 7 under 3, ſo that 53 may ſtand over it, and not 3 only. The Numbers thus placed, ask how often 7 is in 53, the Multiplication Table will help you to find it as before. Having found it out to be 7, I write it with the other Quotient at the right hand, then ſubſtract the Product of 7 the Diviſor by 7 the Quotient, *viz.* 49 from the number placed over it 53, then write the remainder 4 over 3, then daſh out 53 and 7 as in the Scheme. Laſtly, move the Diviſor one ſtep further yet towards the right hand; Then ask by the help of the Table how often 7 is in 42, you will find it to be contained 6 times, therefore placing 6 in the half moon after 7, multiply it by the Diviſor 7, and the Product 42 ſubſtract from the number placed over it 42, and there remains 0; Therefore daſhing out 42 and 7 the work is done, for there is never a figure of the Dividend left, to which the Diviſor may be mov'd, when 'tis come to the laſt, *viz.* 2. So that by this Diviſion you have 976 for the Quotient.

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 54
6832 (97
 77
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 54
6832 (976
 777
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Demonstration.

Becauſe Diviſion is a thing of great concernment and weight in Arithmetick, I ſhall endeavour to make it clearly appear to the Reader, how that the Quotient D found according to the method taught by the Rule, as often contains 1 as the dividend A contains the diviſor. To which end I would have the Reader conſider the particular ſchemes or diagrams of the preceding Operation. In the firſt ſcheme enquiry was made how oft the diviſor 7 might be contain'd in the number 68 ſtanding over it, and 'twas found to be contained in it 9 times, and by that means the figure 9 came to be placed at D for the firſt figure of the Quotient. Now if

A 6832	D 9
B 7	

it were asked if the ſingle number 68 (the following figures 32 being quite taken away) were to be divided by the digit B 7, whether the figure 9 were rightly placed for the quotient, and the number 68 artfully divided by the figure 7, the anſwer would be yes. For the firſt Figure of the Quotient D 9, by the number of its units declares how oft the diviſor 7 is contained in the dividend 68,

68, or (which comes all to one) how often the dividend contains the divisor. So that if the figure 7 be taken or added to it self as often as there are unities in 9, that is, if the figure 7 be drawn into 9, (which is one of the Precepts to be always observed in Division) if the number 68 be not produced, at least the next less than it is, to wit 63, a number made up of 7 added 9 times to it self, but less than 68 by five units; which figure 5 is kept for the second bout being placed over 8.

Now if as well the number 63 containing the divisor 7 exactly nine times, as the Quotient 9 be multiplyed by 100, that so we may have the numbers 6300 and 900: The number 6300 will again as oft contain 7 as there are unities in 900. The reason whereof is plain; For as often as by this Multiplication the number 63 is added to it self, so oft also is the Quotient 9 added to it self, that is, so many unities which intimate how often the divisor 7 is contained in the dividend 68. And therefore since the divisor 7 is found a hundred times oftner in 63 a hundred times repeated, that is, in 6300 than in the single number 63, and it is found nine times in the single number; if that 9 (which is the figure of the Quotient) be a hundred

times repeated, the product 900 will have as many unities as 6300 contains numbers equal to the diviſor 7: Therefore by the definition of diviſion 6300 being divided by 7 the Quotient muſt be 900.

VVhich things being premiſed, ſince that ſame part 63 of the number A 6832 (taking 5 afterward, the exceſs of 68 above 63) belongs to the place of hundreds by reaſon of the two following figures 3 and 2. which poſſeſs the firſt and ſecond places of the number A, it muſt therefore be thought to be 6300; The Quotient alſo 9 belongs to the ſame place of hundreds, by reaſon of two figures more that are to be joyned to it in the proceſs of the work, ariſing from twice moving the diviſor forward, which afterward is to be placed under the figures 3 and 2 of the dividend, ſo that you are alſo to look upon 9 the figure of the Quotient as 900; 'Tis manifeſt that as well 63 as 9 ought to be multiplyed by 100; and therefore (as is already proved) that 6300 as oft contains the diviſor 7 as 900 belonging to the Quotient contains a unit.

A	6832	
		D 9
B	7	

Again, the diviſor 7 being moved one ſtep forward that it might ſtand under the

figure

figure 3 of the dividend, to the end that 53 (uſing 5 or 500 the remaining figure after the firſt work was over) might be placed over the diviſor 7, the diviſor 7 was found to be contained ſeven times in 53, and therefore 7 was put after 9 as the ſecond figure of the Quotient. It will therefore be proved juſt as before, that as well 49 as 7 is drawn into 10, ſo that 49 muſt be look'd upon to be 490, and 7 to be 70, becauſe a ſingle figure follows each of them. From whence as before 'twill follow, that the dividend 490 contains the diviſor 7 as often as 70 belonging to the Quotient contains unity. If therefore 6300 contains the diviſor 7 ſo oft as 900 in the Quotient contains unity, as was ſhewn before, and alſo 490 contains the ſame diviſor 7 as oft as 70 belonging to the Quotient does unity, both numbers 3600, 490 together, that is, their ſum 6790 will as oft contain the diviſor 7 as both numbers 900, 70 together, or their ſum 970 contains unity.

	5	
A	6832	D 97
B	77	

Laſtly, Becauſe, the diviſor 7 being placed (one ſtep forwarder) under the laſt figure of the dividend 2, 42 ſtanding over 7

(for

(for 4 remained when the former work was over, being placed over 3) was found as oft to contain the diviſor 7 as 6 clapt in the quotient did unity. If 42 be added to 6300, 490 (or 6790 their ſum) you will have the dividend 6832, which ſhall contain the diviſor 7 as oft as 900, 70, 6 belonging to the quotient (which added together give 976 the number of the quotient) contain unity. Therefore the quotient 976 was artfully found according to the definition of diviſion, which was the thing to be done.

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  84
A 6832 |
------ | D 976
B  777
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I ſhall here ſhew the Reader how to divide after another manner, which differs nothing from the former, but as to ordering and placing the figures. However 'tis far better than that, becauſe here no figures are daſhed out, ſo that if any error ſhould be committed in the work, the Learner may quickly find out where it is, and after the work is done, the proof of it is both pleaſant and eaſie.

(1.) *Ex-*

(1.) *Example.*

A 6837	B 7
C 63	D 976
——	——
E — 53	C 63
F — 49	F — 49
——	
G — 47	L — 42
L — 42	P — 5
—	——
P — 5	A 6837

To divide 6837 by 7, place them as in the Scheme, then fancy the divisor 7 to have its due place under the dividend, *viz.* to be placed under its figure 8, according to the admonition before given, then put a prick under the figure 8; Then follow the same Rules before made use of. First, Ask how often 7 is in 68, by the way before laid down you will find it to be 9; therefore write 9 after D, just under the divisor B for the quotient. Multiply 9 by 7 the divisor, the product 63 (minding the places) write as well under the quotient D, as under the figure 68 of the dividend. Substract the product thus placed from 68, the remainder 5 place at E in its proper place right under 8; Then to the remainder 5 put 3 the next figure of the dividend, that so you may have 53 to be divided in the next place by 7, which you must imagine to be wrote under

under it as before; Then ask how often 7 is in 53, you will find it to be contained 7 times, therefore place 7 at the quotient D, multiply it by the divisor B, and write the product F 49 both under the quotient, so as 9 may be placed just under 7, and also under the number E 53, then substract this product F from E, writing the remainder G 4 underneath. Clap the following and last figure 7 of the number A to the remainder G 4, that so you may have the number 47 to be divided by 7, which is contained in it 6 times, therefore I place 6 at the quotient D, now 6 multiplying 7 produces L 42, which is wrote under the quotient D, 2 being placed just under 6, and under the number G 47, this product being substracted from 47, the remainder is P 5. Thus the division of the number A by the divisor B is performed, and the quotient found D, no figure being dashed out, all the numbers made by the several Multiplications and Substractions being kept whole and entire, so that if occasion be they may be run over again with ease.

Now to prove whether the work be rightly done do thus, cast 9 out of A the dividend, as often as may be, and clap the re-

remaining figure 6 over the cross, as in the scheme, then cast 9 out of B 7 the divisor, but here because the divisor is a single figure 9 cannot be cast out of it, therefore 'tis to be supposed a remainder after the casting away of 9, so I write this

A 6837 ———— 6

B 7. D 976 7 X 4

C 5 ———— 6

remainder 7 on the left hand of the cross, then cast 9 out of the quotient D as often as you can, and the remainder 4 write on the right hand of the cross. Multiply 7 into 4, to the product 28 annex add 5 the remaining number after the division of A by B was over, out of this number 285 cast away 9, and if 6 remains, to wit, the same that was left when 9 was cast out of the dividend A, the work is rightly performed.

Before I come to instruct the Reader how to divide by a divisor consisting of two, three or more figures, it is necessary that I put him in mind that in choosing the quotient regard is to be had not only to the first or second figure of the divisor, but to all, for although the first and second figure may be such a number of times in the figures of the dividend placed over them,

them, yet if the other be not as oft contained in thoſe over them, I muſt not take that number for my quotient. But as to the choice of the quotient I would have the Reader take care, firſt that the figure he pitches upon be ſuch an one, that multiplying the diviſor the product may be leſs than the figures of the dividend whom it ſtands under, for ſhould it be greater, the diviſor is not ſo often contained in the figures of the dividend placed over it as unity is contained in the quotient, ſince that the number produced by multiplying the quotient into the diviſor contains as often the diviſor, as the number multiplying does unity. Therefore it contains the diviſor oftner than the number placed over it does, which ought to be divided, which muſt not be, ſince it thwarts the nature of diviſion, and overturns the definition lately given of it. Secondly that the figure be ſuch a one that multiplying the diviſor the product may be ſubſtracted from the figures of the dividend ſo as the remainder be not greater than the diviſor, otherwiſe the diviſor would not be ſo often in the figures of the dividend placed over it as unity is in the quotient, which argues the quotient choſen is too little. So that the right quotient is al-

ways

ways ſuch a figure that multiplying the diviſor, the product may be ſuch that it may be ſubſtracted from the figures of the dividend placed over it, and if any remainder be, it be leſs than the diviſor.

(4.) *Example.*

Now to divide 8670 by 34. Firſt, I place 34 under 8670. Secondly, I ſay 3 is twice in 8, and I write 2 in the half moon for the quotient, then I ſay 2 times 3 is 6, 6 from 8 the reſidue is 2, I daſh out 8 and 3 as uſeleſs and write 2

	1
2	28
8670 (2	8670 (2
34	34

over 8, again 2 times 4 is 8, which I ſubſtract from 26, the remainder 18 I write over 26 and daſh 26 and 4. Thirdly, I move the figures of the diviſor one ſtep forwards towards the right hand writing 34 under 87, then I ſay 6 is 3 times in 18, but if I ſhould make 6 the quotient, it multiplying the diviſor would produce a number greater than the figures of the dividend under which 'tis placed, upon which account I ſay 6 is too great, therefore I pitch upon a leſs, *viz.* 5, which I make the ſecond

	1
1	13
13	18
28	28
8670(25	8670(25
344	344
3	3

cond figure of the quotient placing it after 2, then I say 5 times 3 is 15, which substracted from 18 leaves 3, and I write 3 over 8 dashing out 18, and 3 the first figure of the divisor, again 5 times 4 is 20, which from 37 there remains 17, I dash out 3 just now wrote over 8 and 4 the last figure of the divisor writing the first figure of the remainder 17, *viz.* 1 over 3, letting 7 stand in the dividend as it did. Fourthly, I move each cypher of the dividend one step more towards the right hand writing it under 70,

1
13
18
282
8670(255
3444
33

and I say 3 is 5 times in 17, and I write 5 in the half moon then I say 5 times 3 is 15, which from 17 leaves 2 for a remainder, which I write over 7, dashing out 17 and 3, again 5 times 4 is 20, which from 20 leaves 0, so I know 255 is the quotient sought.

(4.) *Example.*

To divide 24096 by 48, first I place 48 under 24096, writing 4 under 24 and 8 under 0, secondly, I say to my self 4 is 6 times

times in 24, but I know preſently that the quotient 6 is too great, becauſe this quotient multiplying the diviſor produces a number that exceeds the figures of the dividend under which it is placed, therefore I pitch upon one leſs, *viz.* 5 for my quotient, and I ſay to my ſelf or in my

$$\begin{array}{l|l} \cancel{2}4096(5 & \cancel{2}\cancel{4}\cancel{0}96(5 \\ \;\;48 & \;\;\cancel{4}\cancel{8} \end{array}$$

mind, 5 times 4 is 20, which ſubſtracted from 24 the reſidue is 4, which I leave where it was in the dividend, daſhing out 2 and 4 the firſt figure of the diviſor, again 5 times 8 is 40, which from 40 leaves 0, and I daſh out 40 and 8, thirdly, I move each figure of the diviſor one ſtep forward towards the right hand, writing them under 09, then I ſay in my mind 4 is never a time in 0 under which I placed it, therefore I write 0 in the half moon.

$$\begin{array}{l|l} & \;\;\;\;\;\;1 \\ \cancel{2}\cancel{4}\cancel{0}96(50 & \cancel{2}\cancel{4}\cancel{0}\cancel{9}6(502 \\ \;\;\cancel{4}88 & \;\;\cancel{4}\cancel{8}\cancel{8}8 \\ \;\;\;\cancel{4} & \;\;\;\cancel{4}\cancel{4} \end{array}$$

I move again each figure of the diviſor forward one ſtep writing them under 96, and I ſay to my ſelf by thought 4 is 2 times in 9, and I write 2 in the half moon, then I ſay 2 times 4 is 8 which from 9 leaves 1 which I write over 9 cancelling 9 and 4. Again 2 times 8 is 16 which from

16

16 leaves 0, by which I know 502 is the quotient ſought.

(5.) *Example.*

To divide 47568 by 395. Firſt, becauſe the firſt figure toward the left hand is greater than the firſt figure of the diviſor, therefore I place the firſt figure of the diviſor exactly under the firſt figure of the dividend, *viz.* 3 under 7. Secondly, I ask my ſelf how oft 2 is in 7, and 'tis manifeſt that it contains it twice; but becauſe I am to have regard to the other Figures of the diviſor alſo, if I ſhould make 2 my quotient, and then go to multiplying each figure of the diviſor by it, and ſo fall to ſubſtracting, I ſhould ſoon meet with a ſubtrahend greater than the number from which I am to ſubſtract it, which thing tells me 2 is too big, therefore I pitch upon 1 for my quotient, then I multiply 3 the firſt figure of the diviſor by 1 the quotient, the product 3 I ſubſtract from 7 the firſt figure of the dividend, the remainder 4 I write over 7, cancelling as well 7 of the dividend as 3 of the diviſor, ſo that the number placed

  4
A ~~7~~568)
————— | C 1
B ~~3~~95 |

over

over 9 the second figure of the divisor is 45 ; which being done I multiply (as before) 9 the second figure of the divisor by 1, and I substract the product 9 from 45 standing over it, writing the remainder 36, consisting of 2 figures over 45, which I presently cancel, as also 9 the second figure of the divisor, as in the Scheme. Which being over I fall to Multiplying and Substracting, as I did before, *viz.* I multiply 5 the last figure of the divisor by 1 the quotient, and I substract the product 5 from 366 standing over it, the remainder will be 361, as in the Scheme. Thus we have made one step into the work, and have got the first figure of the quotient, as the rule prescribes. For the quotient 1 multiplyed the whole divisor B one part after another, and the product is also substracted from the number 756 placed over it by parts, or one single product after another, and the remainder 361 is less than the divisor B. And these are all the operations used in Division, which are to be repeated as oft as the divisor is to be mov'd

```
   3
   46
A 7568 | C 1
  -----
B 395
```

```
   3
   461
A 7568 | C 1
  -----
B 395
```

moved forward in order to the getting a new figure for the quotient.

Thirdly, I move the diviſor forward toward the right hand, ſo that each of its figures are one ſtep nearer the right hand than before, as in the ſcheme. So the figures are rightly ordered for the ſecond operation which is to be performed juſt as the former. Therefore I ask my ſelf how oft 3 the firſt figure of the diviſor B moved forward is contained in 36, (for thoſe figures it ſtands under, as is evident from the Scheme) and I find it to be contained in it 12 times, muſt I therefore place 12 at the quotient? I Anſwer no, by no means. For I muſt never put but a ſingle figure for the quotient (the reaſon of which I will ſhew afterwards). But grant (as it here happens) that the figure of the diviſor be contained ſo many times in the figures over it, that a ſingle figure is too little to expreſs it, yet that the greateſt of the figures, *viz.* 9 may be placed there; muſt one therefore put 9 for the ſecond figure of the quotient? I anſwer, not before tryal be made. For if 9 ſhould be placed in the quotient, you

$$\begin{array}{rl} & 3\ \ \ \\ & \cancel{4}61 \\ A & \cancel{7}\cancel{7}\cancel{6}8 \\ \hline B & 3\cancel{9}\cancel{8}5 \\ & 39\ \ \end{array} \Bigg| \; C\ 19$$

you ſhould alſo draw 9 into 3, the product thence ariſing would be 27, which ſubſtracted from 36 leaves 9, ſo that there would be 91, from which the number produced by multiplying 9 the ſecond figure of the quotient, by 9 the ſecond figure of the diviſor, which is 81, may be ſubſtracted, and much more may the product of the ſame 9 belonging to the quotient multiplyed by 5 the laſt figure of the diviſor, be ſubducted from the number over it. After I have thus reaſoned with my ſelf, I may ſafely clap 9 after 1 for the ſecond figure of the quotient, and by that perform the remainder of the work. So I multiply 9 juſt now placed at the quotient, by 3 belonging to the diviſor, and the product 27 I ſubſtract from 36 ſtanding over it, the remainder is 9, which I write over 6 in the ſame rank that 3 the figure multiplyed ſtands in, cancelling the figures 36 from whom the ſubſtraction was made, and alſo the figure 3 belonging to the diviſor, as having done with them. Again, I multiply the figure of the quotient 9 by 9 the ſecond figure

```
    16
   390
  4613
A 7568
------   C 19 36/396
B 3955
   39
```

of

of the divisor and I substract the product 81 from 91 the number plac'd over it, the remainder 10 must be plac'd over the number 91 (the ranks being always carefully observed) then must 91 and 9 belonging to the divisor be cancelled. Lastly I multiply the same 9 at the quotient by the last figure of the divisor 5, and the product 45 I substract from 108, (for that number you must imagine to stand over 5) and the remainder is 63, which is to be wrote after C 19, and under it the divisor, a line being first drawn between them. By which is meant that after the work of Division is over, there are 63 units left to be divided by the divisor 395. Of which we shall largely treat in the following Book.

Demonstration.

No other proof need be brought of this way of dividing, than what was before laid down about dividing any number by a digit or single figure. For those documents that were useful there, are all likewise to be observed here. If any difference there be it arises from a greater number of figures that the divisor consists of; each of which must be drawn into the figure placed at the quotient, and the single products

ducts to be substracted from the number standing over them, the subductions being severally performed. Touching which way of working it may not to some be so evident, how the whole divisor B 395 can be drawn into every figure of the quotient, when those are multiplyed by each figure of the divisor, and the single products are severally substracted from the numbers placed over them, especially when the work of multiplication is performed not beginning at the last figure 5 of the multiplicand B 395, as the customary way of multiplication is, but at the first figure 3, nor is the usual manner of substracting observed, which begins at the last rank first and so onward. To this difficulty it may be answered, that though the manner of working be somewhat different from that before taught and accustomed (the ease, conveniency and speedy dispatch of division requiring it) however no change or alteration arises from such difference of working, but the self-same thing is performed at last, and comes all to one, just as if each figure of the quotient were drawn into the whole divisor at once, and the whole product thence arising were subducted from the numbers standing over it.

For let us but review the later operation of the preceding example, in which the last figure of the quotient 9 ought to be drawn into the divisor B 395, and the product to be substracted from 3618. Now the figure 9 belonging to the quotient, is first of all drawn into the figure 3, that is the number 300 (because two other figures are behind it) and the product is 2700. 'Tis secondly drawn into 9, the second figure of the divisor, that is into 90, and the product is 810; and last of all into 5 and the product is 45. If therefore these 3 products (whose sum makes the number 3555 produc'd by drawing 9 into the whole divisor B 395) be severally subducted from 3618; that is let the first number 2700 be substracted from it; then from the remainder 918 the second number 810, and lastly from the remainder or appendage 108 the third 45, of necessity the same number will be left, which would have remained, if the whole intire number 3555 (the sum of the three several products) had all at once been subducted from 3618. For both ways the same number of unities are deducted from it, since that any whole is equal to all its parts put together. Nor can any inconvenience arise upon the account of substraction,

on, by reaſon the work begins at the furthermoſt rank toward the left hand, and goes onward toward the right. For thoſe nulla's hanging at the firſt number 2700 (by reaſon they are always as many as are the figures of the dividend 3618 that follow its part 36 ſtanding right againſt 27 as in the diagram) remain untoucht, and are kept intire for the next ſubſtraction. And ſo forward it happens be the partial numbers (produced by drawing any figure of the quotient into the diviſor) never ſo many. Since

A	3618	C 19
B	395	
	2700	
	810	
	45	
	3555	

things are thus, ſince I ſay according to the way laid down the partial multiplications, and partial ſubſtractions of the products from the dividend perform the ſame thing, that by a total drawing any figure of the quotient into the whole diviſor, and a total ſubſtraction of that ſame product from the number ſtanding over it would be done, (and diviſion may rightly and orderly be diſpatched by a total multiplication and total ſubſtraction) 'tis manifeſt that this way alſo of dividing a number by a diviſor conſiſting of divers figures is well

done by means of the foregoing Rule: Which was the thing I intended to demonstrate.

Whilst I was working the foregoing Example (a case happening wherein two figures might seem rightly to be placed at once in the half-moon) I affirmed that two figures could never be placed at the quotient, and made promise to give the Reader the reason of it, which I intend here to make good. Let that same case be proposed once again. 'Twas askt how oft in 36 the first figure of the divisor D *viz.* 3 was contained. Now this is certain that if we look upon the figure 3 alone without any regard had to the following figures, it is contained in the number standing over it 36 twelve times, and so 12 was to be placed at the quotient. But because the other figures also of the divisor 9 & 5 are to be regarded, since they as well as 3 are to be multiplied by the figure put in the quotient, that the product may be subducted from the dividend E, than which the product must never be greater as is before declared. Therefore I positively say, that 12 consisting of two figures is so far from being clap'd in the quotient, that not so much as 10 which is less than 12, and the least number that can be expressed

pressed by two figures ought to be put there. The reason is because the number 10 placed at the quotient ought to be drawn into the whole divisor D 395, and the product ought to be substracted from the number standing over it E 3618, whether multiplication and substraction be performed piece-meal, each figure being taken severally or the whole divisor be multiplied, and the intire product substracted, for the thing comes all to one as was before shewn. But if the divisor D 395, be multiplied by 10, that is, if you put a 0 after the 5 (for so you multiply it by 10, as was taught at page 42) the product becomes greater than the number over it 3618. So that 3950 cannot be substracted from it, upon which account 10 placed at the quotient is too big according to what was taught, page 64. Now that 3950 made by drawing 10 into the divisor 395 is greater than 358 is evident from hence, because 361 that remained after the first operation was over, ought to be less than the divisor 395, and that inequality lyes in those three figures of each number, which cannot be taken away by putting any figure whatsoever to both numbers, for the figures so added will appertain to the first rank, whose greater figure

E 3618

D 3950 | C 110

gure is less than 1 that stands in the second rank: Therefore although 8 be added to the lesser number 361, and 0 clapt behind the greater, notwithstanding this number so made 3618 will always be less than 3950. And therefore the quotient will be always bigger than it ought to be if 10 be it; wherefore a number always less than 10 must be set for the quotient. But there's no number less than 10 but what is a digit, because 10 is the least number of all that consists of two figures. Upon which account what was affirmed is manifestly true.

(3.) *Example.*

To divide 2142178 by 352. 1. I place 352 under 2142178 writing it under 142. 2. I say, to my self 3 is 7 times in 21, but if I should make 7 the quotient it multiplying the divisor the product would be greater than the figures of the dividend under which 'tis placed, therefore I make choice of a less, *viz.* 6 which I write in the half moon, and I say 6 times 3 is 18 which substracted from 21 leaves 3 which I place over 1 cancelling 21 and 3, again 6 times 5 is 30 which from 34 leaves 4, I let 4 stand as it did in the dividend, cancelling 3 and 5, again

```
 ·
 330
2142178 (6
 352
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again 6 times 2 is 12 which from 42 leaves 30, which I write over 42 cancelling 42 and 2. Thirdly, I move each figure of 352 one step forward, writing them under 301, and I say to my self 3 is 1 time in 3, but should I make 1 my quotient, I should be forced to substract a greater number from a less, that is the product of 352 multiplied by 1 from 301 placed over it, therefore I write o in the half moon, omitting substraction for this time, because 352 times o is o.

330
2242178 (60
3822
38

Fourthly, I move each figure of 352 the divisor one step forward, writing it under 017, and I say 3 is 10 times in 30, but 10 is too great, for a quotient must be always a single figure, so I take 9 instead of 10, but should I make 9 the quotient, the product of the divisor by 9 would be greater than the number I am to substract it from, therefore I write only 8 in the half-moon, and I say in my thoughts 8 times 3 is 24, which from 30 leaves 6, which I write over o, cancelling 30 and 3, again 8 times 5 is 40,

2
6
33001
2242278 (608
38222
355
3

which from 61 leaves 21, I write 2 over 6 leaving 1 as it was in the dividend, cancelling 6 and 5, again 8 times 2 is 16, which substracted from 217 (for 2 the last figure of the divisor is supposed to stand under those 3 figures of which we gave the Reader notice before) leaves 201, I dispose of these three figures thus, I let 2 alone over 6 as 'twas before, I write the 0 over 1, and 1 over 7, then I dash out 17 and 2. Fifthly, I move each figure of the divisor forward one step, writing it under 018, and I say 3 is 6 times in 20, but this quotient being too great I write only 5 in the half-moon, saying 5 times 3 is 15, which from 20 leaves 5, I dash out 20 and 3 (the figure multiplied, and the figures standing over it) writing 5 over the 0, again 5 times 5 is 25, which from 51 leaves 26, I cancel 51 and 5 writing 26 over 51, again 5 times 2 is 10, which from 268 leaves 258, I dash out 6 and 2, writing 5 over 6 and leaving 2 and 8 as they were. The remainder after division is done is 258, which I write with the quotient writing under

225
656
33001
2142178 (6085 $\frac{2\,5\,8}{3\,5\,2}$
382222
3888
33

under it the divisor, whereby I know that $6085\frac{2}{3}\frac{5}{5}\frac{8}{2}$ is the quotient sought.

But because this way of dividing may seem difficult to a learner, I'le shew him another manner of dividing longer I confess than the before delivered, but much easier and 'tis thus.

First, I place the divisor just as I did before. Secondly, I seek how many times the last (furthermost toward the left hand) figure of the divisor is contained in those placed over it, then I cancel the divisor, but write it in another place by it self, and under it its product multiplied by the quotient just now found. If this product exceed that part of the dividend under which the divisor is placed, I make choice of another figure for the quotient less by one than the former, by which I multiply the divisor, if the product still be greater than the figures of the dividend under which the divisor is placed, I pitch upon another less by one than that just before chosen, by which I multiply the divisor. And I go on after the same manner till I have a product less than that part of the dividend under which the divisor is plac'd, which product I write under the figures of the dividend that are over the divisor, from whom it being substracted, after the

remainder I write those figures of the dividend under which the divisor has not yet been placed. Thirdly, I move each cypher of the divisor one step forward toward the right hand, going over the same work again, I clap the quotient found in the half-moon. Doing after the like manner till the divisor has been placed under all the figures of the dividend I find at last in the half-moon the quotient I sought after. An example or two will make these Rules mighty plain.

(1.) *Example.*

To divide 2142178 by 352. First, I place 352 under 2142178, writing it under 142. Secondly, I say 3 is 7 times in 21, and I cancel the divisor, but I write it apart, and under it its product by the quotient 7 found, this product is 2464 which exceeds 2142, under which the divisor 352 is placed. Knowing therefore that the quotient 7 is too great, I take 6 by which I multiply the divisor 352, the product is 2112, which is less than 2142 under which the divisor is placed, so I write 6 in the half moon for the first figure of my quotient, and I substract 2112 the product of the divisor 352 by the quotient

tient found 6, from 2142 that part of the dividend under which the divisor is plac'd, the remainder is 30, and after this remainder I write 178 the remaining part of the dividend under which the divisor has not yet been placed.

2142178 (6	For the first figure of the quotient.
$\cancel{352}$	352 352
2112	7 6
30 | 178	2464 2112

Thirdly, I move each figure of the divisor one step forward, writing it under 301, and I say 3 is once in 3, but perceiving at first sight that 352 the product of the divisor 352, by the quotient found exceeds 301 under which the divisor is placed, I write 0, less by one than 1 in the half-moon, and I cancel 352, without substracting any thing from the figures standing over the divisor, because 352 by 0 is 0.

2142178

2142178(60 ~~352~~ 2112 30|178 ~~352~~	For the second figure of the quotient. 352 352 7 6 2464 2112

Fourthly, I move one step each figure of the divisor writing it under 017, and I say 3 is 10 times in 30, but (I know full well I must not take a figure greater than 9 for the quotient) so I take 9 at a venture, afterward I cancel the divisor 352, but I write it a part, and under it its product by the quotient 9 just now found, this product is 3168 which exceeds 3017 under which the divisor 352 is placed, therefore I take 8 by which I multiply 352, and the product 2816 not exceeding 3017 I write 8 in the half-moon, and substract 2816 the product of 352 by 8 from 3017 the remainder is 201, after which I write 8, under which the divisor has not yet been placed.

2142178 (608
~~352~~
2112

30|178
~~352~~
~~352~~
2816

201|8

For the first figure of the quotient.

352	352
7	6
3464	2112

For the third figure of the quotient.

352	352
9	8
3168	2816

Fifthly, I move one ſtep each figure of the diviſor writing it under 018, ſaying 3 is 6 times in 20, then I cancel 352, but I write it apart, and under it its product by 6, which is 2112, which exceeds 2018, under which 352 is placed, therefore I take only 5, by which I multiply 352 and the product 1760 being leſs than 2018, I write 5 in the half-moon, and I ſubſtract 1760 the product of 352 by 5 from 2018, the remainder is 258, which I write with the quotient writing under it the diviſor. So the work is done, not ſo ſpeedily I con-
feſs

fess as when we made use of the first way, but however with ten times more ease, so that without doubt this way is far more convenient for a beginner to practise by than the former.

2142178(6085 $\frac{258}{352}$
352
2112

30|178
352
352
2816

201|8
352
1760

258

For the first figure of the quotient.

352	352
7	6
2464	2112

For the third figure of the quotient.

352	352
9	8
3168	2816

For the fourth figure of the quotient.

352	352
6	5
2112	1760

(2.) *Ex-*

(2.) *Example.*

To divide 73394561 by 179. First, I place 179 under 73394561, writing it under 733. Secondly, I say 1 is 7 times in 7, but knowing at first sight 7 is too great, I make choice of 6, and I cancel 179, but I write it apart and under it its product by 6 the quotient found, this product is 1074, which far exceeds 733 under which the divisor 179 is placed, therefore I pitch upon 5, by which I multiply 179 the product is 895 which exceeding 733 under which 179 is placed, I am forced to take but 4, by which once again I multiply 179 and the product 716, not exceeding 723, I write 4 in the half moon, and I substract 716 from 733 the remainder is 17, after which I write 94561.

73394561(4	For the first figure of the quotient.
~~179~~	
716	
17 | 94561	

179	179	179
~~6~~	~~5~~	4
~~1074~~	~~895~~	716

Third-

Thirdly, I move each figure of the divisor one step forward, writing it under 179, and I say 1 is once in 1, and perceiving presently that the product of the divisor by the quotient is 179 which is not greater than 179 plac'd over the divisor, I write 1 in the half-moon, & I substract 179 the product of the divisor 179 by 1 from 179 under which the divisor is placed, and there remains 0.

Fourthly, I move each figure of the divisor one step forward writing it under 004, and perceiving that 1 is never a time in 0, I write 0 in the half-moon, and cancelling the divisor 179 I put it one step forwarder under 045, but seeing also that 1 is never a time in 0 under which I have placed it, I write another 0 in the half-moon, cancelling 179.

Fifthly, I move each figure of the divisor one step more forward writing it under 456, and I say 1 is 4 times in 4, but knowing at first sight that 4 is too great, I take 3 only and I cancel 179, but I write it apart and under it its product by 3, this product is 537, which exceeds 456 under which 179 is placed, therefore I pitch upon 2, by which I multiply 179, and the product 358 being less than 456 I write 2 in the half moon and I substract 358 from 456, the remainder is 98 after which I write 1. 73394561

73394561(41004
~~179~~
716

17|94561
~~179~~
179

000|4561
~~179~~
~~179~~
~~179~~
358

98|1

For the first figure of the quotient.

179	179	179
~~6~~	~~5~~	4
~~1074~~	~~895~~	716

For the fifth figure of the quotient.

179	179
~~3~~	2
~~537~~	358

Lastly, I move each figure of the divisor 179 one step forwarder, writing it under 981, and I say 1 is 9 times in 9, but knowing presently that 9 is too big, I take 8 only, and I cancel 179 but I write it apart, and under it its product by 8, this product is 1432, which exceeds 981, I therefore take only 7, by which I multiply 179, the product is 1253, which also exceeds 981, so I take only 6 by which I multiply 179, the product is 1074, which also exceeding 981, I am constrained once again to

to take but 5, by which also I multiply 179, and the product 895 being less than 981, I write 5 in the half-moon, and I substract 895 the product of 179 by 5 from 981, the remainder 86 I write with the quotient, writing under it the divisor 179. So the work being done I know $410025\frac{86}{179}$ is the quotient sought.

$73394561(410025\frac{86}{179}$

~~179~~

716

17|94561

~~179~~

179

000|4561

~~179~~

~~179~~

~~179~~

358

98|1

~~179~~

895

James

86

For the first figure of the quotient.

179	179	179
~~6~~	~~5~~	4
~~1074~~	~~895~~	716

For the 5th. figure.

179	179
~~3~~	2
~~537~~	358

For the 6th. figure.

179	179	179	179
8	7	6	5
1432	1253	1074	895

I shall

I ſhall ſhew the Reader one way more of dividing, which though it differ very little from the former, yet I think it not amiſs to lay it down, and 'tis thus. To divide 7568 by 395, place them as in the ſcheme, imagin notwithſtanding the diviſor to be placed under the three firſt figures of the dividend, and under the laſt of them clap a prick or point, that ſo I may know how far I have gone the firſt bout. Then I ſeek how oft 3 is in 7, but before I put any thing in the half-moon, I ponder upon it to know whether I may ſafely do it. After tryal I find 3 can be contained but once in 7, therefore I put 1 after C, then I multiply the diviſor B by 1 that the product may be D, which I ſo write under the quotient, that its laſt figure 5 may be directly under 1 the firſt figure of the quotient. Then I write the ſame product under that part of the dividend under which the diviſor according to the firſt method of dividing ſhould have ſtood, ſubſtracting it from 756, and I write the remainder 361 under, after the ordinary way of ſubſtraction. This being done I move each figure of the diviſor one ſtep forward, that is I write 8 (the next figure to that that has a point under) after the remainder making it one number

ber that is next to be divided. Then I fancy the divisor B to be wrote under it just as if it were a new number to be divided, so as its first figure 3 may be just under 6, and the number 36 may be supposed to stand over 3. Then I seek how oft 3 is in 36 supposed to be placed over it, rumbling the business a little in my mind I pitch

A	7568	B	395
D	395	C	19
E	3 18	D	395
F	3555	F	3555
G	0063	G	0063
		A	7568

upon 9 which I make the second figure of the quotient, then by it I multiply the divisor B 395, writing the last figure of the product F 3555 directly under the multiplier 9, which same product write also under E, then substract it from E. The remainder is 63. Thus the work is done, there being never a figure of the dividend left to be added to the residue. If the Learner list to know whether the work be rightly performed let him do thus. Under D and F let him write G as is usual in addition, then let him add these three numbers D, F and G together. The sum of them is the dividend if the work be rightly performed.

The Reader could not choose but take notice

notice that the main difficulty of division lyes in getting a fit quotient, such a one I mean that multiplying the divisor makes a product, that is neither so great as to exceed the figures of the dividend under which the divisor is placed, nor yet so little as to leave a remainder greater then the divisor. Several ways has been invented for removing this difficulty, which when the first way of dividing (where cancelling the figures is part of the work) is put in practice is very great. Of these ways I shall lay down three or four.

The first way.

If the Reader in practising the first way of division chances to pitch upon a quotient too big he may thus rectifie it. Suppose the product made by multiplying the divisor by the quotient exceed the figures of the dividend under which the divisor is placed, tis a certain sign the quotient is too great, therefore I add the divisor to that part of the dividend from whence I should have substracted the product of the divisor by the quotient, had it not been too big, as often as is necessary, that is till I have a number greater than the subtrahend. After which I take away as many unities from the quotient, as the divisor was added times to the figures placed

over

over it. By this means I meet with a fit quotient. One Example will make all this very plain.

Example.

To divide the number A by B. First of all you try how often 1 the first figure of the divisor may be contained in 9 the figure of the dividend right over it. Here the learner will be often put to a plunge, by reason this tryal is several times to be repeated, for the divisor consisting of many figures in choosing his quotient he must have regard to them all. But without more ado ghess as near as you can and suppose 1 to be 5 times in 9, placing 5 in the half-moon, by which multiply 1, substract the product 5 from 9 that is right over it, the residue will be 4, cancel 9 and 1 and write the remaining figure 4 over 9. After that 9 multiplyed by 5 makes the product 45 greater than 42 under which 9 is placed, so that I cannot substract 45 from it. Therefore 5 is too big. The learner finding he has pitched upon a wrong figure for the quotient, doing thus he will meet with a right one that will serve his turn. Let him add to 42 too little for 45 to be substracted from it the divisor 19 as often

as is necessary, that is till 42 exceed 45 the subtrahend. Here once adding it is enough, for by this addition 42 becomes 61, (which I place over 42 as in the scheme) greater than 45. Now substract 45 from 61, the remainder 16 to be placed over 61 is the true number that ought to be left, had the figure placed in the half-moon been the right one Which fit quotient will presently be had, if I take away as many unities from 5 as the divisor was added times to the number placed over it, that so it might become greater than the subtrahend. As here the divisor 19 was added once to 42 which made 61 greater than 45, therefore I take 1 once from 5 that it may become 4, the quotient that ought to be, with which you must continue your work rejecting the former. If the learner had pitcht upon 6 for his quotient and so had gone on with his work, the divisor 19 must have been added twice, that so the Reader might light upon a number greater than the subtrahend, and upon that account two unities must have been taken from 6, the figure placed for the quotient; that so the right figure 4 might be met withal, as

$$\begin{array}{llll} & & 1 & \\ & & \cancel{6}6 & \\ & & \cancel{4}\cancel{1} & \\ A & \cancel{9}\cancel{2} & 1 & \\ \hline B & 19 & & (\cancel{5}4 \end{array}$$

also

alſo the ſame number 16 remaining after ſubſtraction with which the work is to be carried on.

If the diviſor had conſiſted of more than two figures, the laſt of them muſt have been multiplied by the new rectified quotient juſt as if it had been firſt of all made choice of. As in the former Example, had the diviſor been 196 inſtead of 19, the figure 5 being rectified and changed into 4 while the practitioner was buſie about 19, afterwards the remaining figure of the diviſor 6 ought to be multiplied by the new quotient 4, and the product 24 ought to be ſubſtracted from 160 placed over 6 after the uſual way of dividing. If at any time the Reader ſhould have occaſion to divide a great number by another conſiſting of many figures and he would perform his work after the common way by cancelling figures as he goes along, this trick of mending the quotient juſt now taught him will ſtand him in great ſtead.

On the other hand if the Reader pitches upon a figure for the quotient too ſmall, which cannot be perceived till the work is done, ſince the only ſign of its being too little is that the remaining number after the product of the diviſor by that figure is ſub-

gure is ſubſtracted from the number ſtanding over the diviſor is greater than the diviſor, and upon that ſcore may again be divided by the ſame diviſor. To remedy this do thus. When it ſo happens that the remainder (the product of the diviſor by the quotient being ſubſtracted from the dividend over it) is greater then the diviſor, in this caſe let not the learner move the diviſor in order to begin a ſecond operation, *i. e.* to find a ſecond figure to be placed in the half-moon, for that he was about is not yet finiſhed ; but let him divide the remaining number that proves greater than the diviſor, by the ſame diviſor as it ſtands, juſt as if he were beginning a new work. However he muſt take notice that as the diviſor is not moved forward, ſo neither is the ſecond quotient to be placed one rank beyond the firſt,but in the ſame rank with it, that of two they may become one, as proceeding from the ſame operation had it been rightly performed. An Example will make this plain.

As for Example.

The number A is to be divided by the number B. One muſt ſeek firſt of all how

oft 1 the first figure of the divisor is contained in 9 placed over it, suppose it to be contained thrice, place 3 at C for the quotient, by the help of which the necessary multiplications and substractions being performed, the remaining number is 35, which exceeds the divisor 19, therefore divide again 35 by 19, that is try how oft 1 the first figure of the divisor is contained in the figure 3 placed over it, imagin that 'tis once in it; therefore 1 is to be placed at the quotient, not just after 3 that's there already, but in the same rank with it, *viz.* under as in the scheme, that by adding them, out may come 4, this last work being done, all the numbers are restored, both the quotient 4 and also the remaining number 16 placed over 25, which is less than the divisor 19, so that now the same divisor may be moved forward in order to begin the following work.

$$\begin{array}{lll} & 1 & \\ & 2 & \\ & 36 & \\ & \cancel{8}5 & \\ A & \cancel{9}\cancel{2}1 & \\ \hline B & \cancel{1}\cancel{9} & (C\ 34 \\ \hline & & \quad\ 1 \end{array}$$

The Reader may take notice that in case the right figure to be placed in the quotient cannot easily be lighted upon, 'twill be better to take a figure too little than one too big, for the former will set things to rights sooner and easier than the other.

other. But take which he will his work will go forwards, for let him ſtray either one way or t' other, the ſleight juſt now taught him will quickly bring him into his way again.

The ſecond way to eaſe the toyl of Diviſion.

This way is the ſame with that laid down when we were about multiplication, though here we make another uſe of it. And before I teach it the Reader I cannot forbear to tell him that when the dividend and diviſor conſiſts of many figures each this method will wonderfully help the young or old practiſer, for in my opinion by this means Diviſion is no harder than addition. The Example I ſhall lay down to illuſtrate the buſineſs is for plainneſs ſake in ſmall numbers, but I would have the Reader make uſe of it when he has occaſion to work upon great ones.

If therefore the number A were to be divided by B, let the Reader make a particular Table for the diviſor B after the ſame manner the Multiplication Table was before framed in page 31, and as here in the ſcheme by adding 19 to it ſelf, then to the ſum 38 that it may be 57, and ſo on, placing the nine digits on the

left hand anſwering to theſe ſums. Make uſe of the Table to accompliſh thy intended purpoſe thus. The number 19 ſhould

A 9217	B —— 19	C 485
76	1 —— 19	
161	2 —— 38	
152	3 —— 57	
———	4 —— 76	
0097	5 —— 95	
95	6 —— 114	
———	7 —— 133	
000 | 2	8 —— 152	
	9 —— 171	

be written under 92, therefore ſeek in the Table for the next leſs number than 92, this number is 76, over againſt which on the left hand ſtands 4, therefore make 4 the quotient, then ſubſtract 76 from 92 that the remainder may be 16, to this number adding the following figure 1 of the dividend A (for by this means the diviſor is moved forwards) ſo you have the number 161 to be divided the next bout. Seek therefore in the Table either for 161 or the number next leſs than it, and that number is 152. over againſt which is placed 8, which is to be the ſecond figure of the quotient, then the number 152 being ſub-

substracted from 161 the remainder is 9, close to which I place 7 the last figure of the dividend A, so I have 97 which comes next to be divided. Therefore this number or the next less than it being sought in the Table you find 95, against which stands 5, which is to be the third figure of the quotient, then subtract 95 from 97 the remainder is 2. Thus you have done your work with as much ease and pleasure as can be desired.

A third way of shunning the difficulty of Division.

When the divisor chances to be a compound number (*i. e.* such a one that another number besides unities can divide) which may be produced by multiplying two other numbers one into another, as 24 which is gotten by the numbers 6 and 4 multiplyed into each other. Then the dividend may be divided by either of these two numbers, the quotient arising from that division may again be divided by the other number, the quotient of this last division is the self-same with that the propounded number divided by the whole divisor would produce. For instance, were the number A to be divided by B, the quo-

A 768

B 24 (C 32

A 768

D 6 (E 128

F 4 (C 32

tient would be 32. Which quotient would be made, were the same number A divided by 6 and the quotient thence arising E 128 were again divided by F 4.

A fourth way of easing Division.

Whensoever the divisor has one or more nulla's for its last figures, the work of division is performed, if you cast away those noughts and as many figures of the dividend, and then divide the remainder of the dividend by that of the divisor, if any thing remain after division is over, joyn to it those figures before cast away, all which write close after the last figure of the quotient with the whole divisor under it fraction-wise.

A 47894

B 2500 (C $19\frac{3\,94}{25}$

As suppose the number A were to be divided by B, cast away the two last figures of A 94, and the remaining number 478 divide by 25 which is the divisor, the two nulla's being

ing discarded, the quotient is C, 3 remaining, to which the rejected figures of the dividend being added, I write the whole close after the quotient with the divisor under. Hence one may easily divide any number by another that has several nulla's before which is prefixed a unit or 1. As suppose by 10, 100, &c, For you need do no more but cast away as many figures from the dividend as the divisor has nulla's, make the rest the quotient, close after which write the rejected figures with the divisor under them. As to divide 4983 by 10, I cast away the last figure of the dividend, the divisor having but one nulla, the rest of the figures are the quotient, after which I write the figure cast away with the divisor under it thus, $498\frac{3}{10}$, had the divisor been 100 it had stood thus, $49\frac{83}{100}$.

If the Reader list at any time to make tryal of his work whether it be rightly performed or not; he need do no more but multiply the divisor by the quotient, if the product be the same number with the dividend, no doubt but the work is artfully performed. However I would advise the Reader to be careful in his multiplication for fear of committing an error, which if committed would lead him into an opini-

on that he was out in his division though exactly done.

The Use of Division.

If one would at any time reduce any thing of a less denomination to another thing of a greater,'tis but dividing the lesser by the number intimating how many times the less is contained in the greater, and the business is done. As to reduce 72 pence to shillings, I divide 72 by 12 intimating how often a peny is in a shilling, and the quotient is 6 shillings. So to reduce 36 feet into yards, I divide 36 by 3 the quotient 12 is 12 yards.

Moreover, this question and the like may be solved by division alone. Ten men had 5 pound given them, what must each man have for his share? the 5 pound reduced to shillings, by Rule in page 49 is 100 shillings, which divided by 10 the number of the men is 10 shillings, the share each man is to have. This question and such like may be performed also by the Rule of Three thus, if 10 men give 100 shillings what does 1 give? the fourth term of this proportion is each mans share.

THE

THE SECOND

BOOK

OF

ARITHMETICK:

SHEWING

How to work upon Broken Numbers, as the former did upon Whole Numbers.

HAving ſhewn the Reader the four uſual ways of working upon whole Numbers, I come in the next place to teach him how to handle Fractions when they fall in his way. Which broken numbers however at firſt they may ſeem obſcure to the learner, yet after he has ſometime tampered with them they'l become familiar to him and for the moſt

part very easie. A broken number therefore is such a one that consists of some of those parts into which unity is divided. Now to express a Fraction or piece of a number two numbers are altogether necessary, the one to intimate into how many parts unity is divided, the other to declare how many of those parts each Fraction contains. The former is commonly called the denominator, or second term of that Fraction, the latter the numerator or its first term. The latter is always the uppermost figure under which (a line being drawn) stands the former thus, $\frac{4}{7}$, which Fraction I express thus, *four sevenths*, the Fraction $\frac{15}{32}$ thus, *fifteen thirty seconds*, &c. The numerator of a Fraction (properly so call'd) is always less than the denominator, for the numerator does not lay claim to all the parts into which unity is divided, otherwise 'twould be equal to a whole number, and consequently to be esteemed as such and not a Fraction. However it often happens that the numerator is greater than the denominator, and such a Fraction is called an improper fraction as $\frac{3}{2}$, yet you may deal with such as with those that are proper fractions, they having the same qualities and being subject to the same laws with the other. Having briefly

briefly explained the nature of a fraction, I shall instruct the Reader in the next place how to manage them, which he will be able sufficiently to do when he can readily perform these seven Problems following.

The first Problem.

To find the greatest common divisor, or measure of any two numbers. Now a common divisor is a number, that divides two other numbers without a rest, as 6 divides 12 and 36. To do the thing required supposing both numbers to be whole ones. First divide the greater by the less. Secondly, If this division leaves yet a remainder or a fraction, divide the second term of the fraction by the first. Thirdly, If this division leaves yet a remainder, divide the second term by the first, these divisions are to be repeated till you come to one that leaves no rest. The divisor of this division is the divisor you want. The Examples will make all this plain.

(1.) *Example.*

To find the greatest common divisor of 32 and 64, first divide 64 by 32 the quotient

tient is 2, and by reaſon this diviſion is performed without any remainder, 32 the diviſor of this ſame diviſion is the greateſt common diviſor of 64 and 32, for 32 cannot have a greater diviſor than it ſelf.

$\cancel{6}\cancel{4}$ (2
$\cancel{3}\cancel{2}$ 32 the diviſor ſought

(2.) *Example.*

To find the greateſt common diviſor of 30 & 25, firſt divide 30 by 25 & the quotient is $1\frac{5}{25}$, ſecondly, this diviſion leaving a fraction $\frac{5}{25}$ divide 25 the ſecond term of the ſame fraction by 5 which is its firſt term, and becauſe this diviſion leaves no remainder, 5 the diviſor of this ſame diviſion is alſo the greateſt common diviſor of 30 and 25.

firſt diviſion.	ſecond diviſion.
$3\cancel{0}$ ($1\frac{5}{25}$	$2\cancel{5}$ (5
$2\cancel{5}$	$\cancel{5}$
	5 the divi. ſought

(3.) *Example.*

To find the greateſt common diviſor of 27 and 21, firſt divide 27 by 21, and the quo-

quotient is $1\frac{6}{21}$, secondly this division leaving a remainder or fraction $\frac{6}{21}$, divide 21 its second term by the first which is 6, the quotient is $3\frac{3}{6}$. This division leaving also a fraction $\frac{3}{6}$ divide 6 by 3, the quotient is 2, and because this division leaves no rest, 3 which is its divisor is also the greatest common divisor of 27 and 21.

1st. division.	2d. division.	3d. division.
		$\cancel{6}$ (2
$\cancel{27}$ ($1\frac{6}{21}$	$\cancel{21}$ ($3\frac{3}{6}$	3
$\cancel{21}$	$\cancel{6}$	3 the divisor sought

(4.) *Example.*

To find the greatest common divisor of 98 and 47, first divide 98 by 47, and the quotient is $2\frac{4}{47}$, secondly, this first division leaving a fraction $\frac{4}{47}$ divide 47 by 4, and the quotient is $11\frac{3}{4}$. This second division leaving the fraction $\frac{3}{4}$ divide 4 by 3 and the quotient is $1\frac{1}{3}$, fourthly divide 3 by 1 the quotient is 3, and because this division leaves no remainder, its divisor is also the greatest common divisor of 98 and 47.

1st. di-

1st. divis.	2d. divis.	3d. divis.	4th. division.
~~98~~ (2 $\frac{4}{47}$	~~47~~ (1 1 $\frac{3}{4}$	~~4~~ (1 $\frac{1}{3}$	~~3~~ (3
~~47~~	~~44~~	~~3~~	~~1~~ 1 the div. sought

The second Problem.

To find the quotient (a number that declares how often the first term of a fraction contains the 2d, or is contained in it, as the quotient of $\frac{8}{4}$ is $\frac{2}{1}$ or 2, of $\frac{4}{8}$ is $\frac{1}{2}$, 1 being contained as often in 2 as 4 is in 8) of any fraction proposed or in other words to reduce any fraction to its least terms, first find out the greatest common divisor of the two terms of the fraction by the foregoing Problem. Secondly, make the first quotient the first term, the second the second term of a fraction which is the quotient sought. An Example or two will make all plain.

(1.) *Example.*

To find the quotient of $\frac{56}{56}$, first I divide each of the terms by 56 their greatest common divisor. Secondly, I make 1 the first quotient of these divisions the first term, and

and 1 the second quotient the second term of the fraction $\frac{1}{1}$ or 1 which is the quotient sought.

1st. division	1st. quoti.	2d. division	2d. quotient
56	(1	56	(1
56		56	

$\frac{1}{1}$ the quotient sought.

(2.) *Example.*

To find the quotient of $\frac{64}{32}$, first divide each of the terms 64 and 32 by 32 which is their greatest common divisor. Secondly make 2 the first quotient of these divisions the first term, and 1 the second quotient the second term of the fraction $\frac{2}{1}$ or 2 which is the quotient sought.

1st. division	1st. quoti.	2d. division	2d. quoti.
64	(2	32	($\frac{1}{1}$
32		32	

the quotient sought $\frac{2}{1}$

(3.) *Ex-*

(3.) *Example.*

To find the quotient of $\frac{378}{1524}$, first I divide each of the terms 378 and 1524 by 6 their greatest common divisor. Secondly, I make 123 the first quotient of these divisions the first term and 254 the second quotient the second term of the fraction $\frac{123}{254}$, which is the quotient sought.

1st. division.	1st. quotient	2d. division.	2d quotient
378	(123	1524	(254
~~6~~		6	

$\frac{123}{254}$ the quotient soug.

Whensoever the Reader has occasion he may reduce a whole number to a fraction by writing 1 under, as to reduce 6 I write it thus $\frac{6}{1}$, whereby I have not altered the value of 6, for 1 whether it multiply or divide another number does not at all alter it.

To reduce a whole number to any fraction proposed the Reader may do thus. Let

Let him multiply the integer by the denominator of the fraction to which he intends to reduce the whole number, let him make the product the numerator, under which let him write the denominator of the proposed fraction. Thus 7 may be reduced to fifths by multiplying 7 by 5, and under the product 35 writing 5, that it may become this fraction $\frac{35}{5}$. If to a whole number there adheres a fraction, you may turn the whole number and fraction into another fraction of the same denomination after the same manner, only to the product you must add the numerator of the fraction. As to reduce $8\frac{2}{5}$ to fifths, multiply 8 by 5, to the product add 2, so you have the fraction $\frac{42}{5}$.

I suppose the Reader can already add and substract fractions that have the same second term. For example that the fractions $\frac{1}{4}$ and $\frac{2}{4}$ added make $\frac{3}{4}$, and on the contrary that $\frac{1}{4}$ substracted from $\frac{3}{4}$ leaves $\frac{2}{4}$.

An Advertisement.

The Reader may take notice that it is very necessary that he reduce his fractions to their least terms, before he goes to work upon them after the several ways hereafter laid down. For by this means they will

will become much more intelligible, and the working of them will be shorter and easier.

The third Problem.

To cause two fractions that have different second terms to have each of them the same without altering their value.

If the lesser second term can divide the greater precisely, take the quotient and by it multiply each term of the fraction which has the lesser second term, and the work is done.

But if the lesser second term cannot so divide the greater, multiply the first term of the first fraction by the second term of the second fraction, and the first term of the second fraction, by the second term of the first, these two products are to be the numerators; the product of the two second terms being denominator to both.

(1.) *Example.*

To reduce 6 or $\frac{6}{1}$ and $\frac{9}{4}$ to the same denomination, the quotient of the greater second term divided by the lesser is 4, by which I multiply each term of $\frac{6}{1}$ whose second

cond term is least, so I have $\frac{24}{4}$ equal to $\frac{6}{1}$, | $\frac{6}{1}$ and $\frac{9}{4}$ equal to $\frac{24}{4}$ and $\frac{9}{4}$

thus instead of $\frac{6}{1}$ and $\frac{9}{4}$ I meet with $\frac{24}{4}$ and $\frac{9}{4}$, which without any alteration of their value have the same second term.

(2.) *Example.*

To give the same second term to $\frac{3}{2}$ and $\frac{5}{6}$, 6 divided by 2 the quotient is 3 by which I multiply each term of $\frac{3}{2}$, which gives me $\frac{9}{6}$ equal to $\frac{3}{2}$, so instead of $\frac{3}{2}$ and $\frac{5}{6}$ | $\frac{3}{2}$ and $\frac{5}{6}$ equal to $\frac{9}{6}$ and $\frac{5}{6}$

I have $\frac{9}{6}$ and $\frac{5}{6}$, which have the same second term their value remaining unalter'd.

(3.) *Example.*

To give the same second term to $\frac{5}{4}$ and $\frac{5}{6}$, 4 can't divide 6 precisely, therefore I multiply 5 (the numerator of the first fraction) by 6 (the denominator of the second) the product is to be the numerator of a fraction equal to $\frac{5}{4}$, then I multiply 5 (the numerator of the second fraction) by 4 the denominator of the first, of the product I make a numerator of a fraction that is to be equal to $\frac{5}{6}$, lastly I make the pro-

duct

$\frac{5}{4}$ and $\frac{5}{6}$ equal to $\frac{30}{24}$ and $\frac{20}{24}$. duct of 6 by 4 the denominator to both, This gives me $\frac{30}{24}$ equal to $\frac{5}{4}$ and $\frac{20}{24}$ equal to $\frac{5}{6}$, so that in stead of $\frac{5}{4}$ and $\frac{5}{6}$, I have $\frac{30}{24}$ and $\frac{20}{24}$, which have the same second term their values remaining unchanged.

The fourth Problem.

To add two or more fractions.

First by the former Problem reduce the two first fractions to the same denomination, which done add them together, reduce their sum (by the second Problem) to its least terms.

Secondly, Give the same second term to the sum thus reduced and to the third fraction, then add them together, and so on if there be more.

Example.

To add $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{5}$ into one sum, first by the preceding Problem you find $\frac{2}{3}$ and $\frac{5}{6}$ to be equal to $\frac{4}{6}$ and $\frac{5}{6}$ whose sum is $\frac{9}{6}$, the quotient of it is $\frac{3}{2}$. Secondly, you find $\frac{3}{2}$ and $\frac{7}{5}$ to be equal to $\frac{15}{10}$ and $\frac{14}{10}$ whose sum is $\frac{29}{10}$, which being equal to $\frac{2}{3}$ more $\frac{5}{6}$, more $\frac{7}{5}$, 'tis the sum desired.

When

When ſome of the fractions to be added happen to be improper, get out of them what they contain that is equal to a whole number, writing the reſt fraction-wiſe.

Example.

To add $\frac{5}{4}$, $\frac{7}{3}$, $\frac{23}{5}$ and $\frac{1}{2}$, firſt I ſay $\frac{5}{4}$ is equal to $1\frac{1}{4}$, $\frac{7}{3}$ is equal to $2\frac{1}{3}$ and $\frac{23}{5}$ is equal to $4\frac{3}{5}$. Then I add $\frac{1}{4}$ and $\frac{1}{3}$ whoſe ſum is $\frac{7}{12}$, to which I add $\frac{3}{5}$, the ſum is $\frac{71}{60}$ equal to $1\frac{11}{60}$, I ſet 1 aſide adding $\frac{11}{60}$ and $\frac{1}{2}$ the only proper fraction, the ſum is $\frac{41}{60}$, to which I add the four whole numbers, 1, 2, 4, and 1 ſet aſide juſt before, and the ſum total is $8\frac{41}{60}$ the ſum ſought.

The fifth Problem.

To ſubſtract fractions one from another.

Firſt add the fractions to be ſubſtracted into one ſum (by the preceding problem) then add together thoſe from whom you would ſubſtract them, after that reduce the two ſums to the ſame denomination.

Secondly, Subſtract the laſt ſum from that foregoing, the remainder is that you ſeek after.

Ex-

Example.

To ſubſtract $\frac{2}{3}$ and $\frac{5}{6}$ from $\frac{3}{4}$ and $\frac{6}{7}$, the ſum of $\frac{2}{3}$ and $\frac{5}{6}$ is $\frac{9}{6}$, which by the ſecond Problem is equal to $\frac{3}{2}$, that of $\frac{3}{4}$ and $\frac{6}{7}$ is $\frac{45}{28}$, giving the ſame ſecond term to $\frac{3}{2}$ and $\frac{45}{28}$, you find $\frac{3}{2}$ to be equal to $\frac{42}{28}$, which being ſubſtracted from $\frac{45}{28}$, the remainder is $\frac{3}{28}$ for the difference ſought.

When the fractions chance to be improper I would adviſe the Reader to go this way to work, firſt of all let him take out of them what they contain equal to unity or any other whole number, afterward if there remain a fraction in the ſubtrahend, ſubſtract this fraction from that remaining in the other, if ever a greater be there otherwiſe from 1 taken from the whole number; the remainder thence ariſing I add to the fraction too little for ſubſtraction; the ſum of them I add to the remainder after the leſſer whole number is ſubſtracted from the greater.

(1.) *Example.*

To ſubſtract $\frac{19}{14}$ from $\frac{7}{2}$, firſt I ſay $\frac{19}{14}$ is equal $1\frac{5}{14}$ and $\frac{7}{2}$ is equal to $3\frac{1}{2}$. Secondly, I ſubtract $\frac{5}{14}$ from $\frac{1}{2}$ or $\frac{7}{14}$ that is equal to it,

it, and the remainder is $\frac{2}{14}$ or $\frac{1}{7}$ (by the second Problem.) Afterwards I substract 1 from 3, to the remainder 2 I add the residue $\frac{1}{7}$ which makes 2 $\frac{1}{7}$ the difference sought.

(2.) *Example.*

To substract $\frac{26}{3}$ from $\frac{846}{10}$, first I say $\frac{26}{3}$ is equal to 8$\frac{2}{3}$ and $\frac{846}{10}$ is equal to 84$\frac{3}{5}$. Secondly, I perceive that $\frac{2}{3}$ or $\frac{10}{15}$ its equal cannot be substracted from $\frac{3}{5}$ or $\frac{9}{15}$, upon this account I substract $\frac{2}{3}$ from 1 taken from 84, writing 83 instead of it, the remainder is $\frac{1}{3}$. In the next place I substract 8 from 8, the residue is 75. Then I add $\frac{1}{3}$ or $\frac{5}{15}$ to $\frac{3}{5}$ or $\frac{9}{15}$ which makes $\frac{14}{15}$, which last of all I add to 75, and 75$\frac{14}{15}$ is the remainder sought.

'Tis often of great importance to know how much one fraction is greater or less than another. Which may easily be known if you reduce them to the same denomination and afterward substract one from the other.

Example.

To know how much $\frac{5}{7}$ is greater or less than $\frac{8}{11}$, you must reduce them to the same

same

ſame ſecond term, by which means you have $\frac{55}{77}$ equal to $\frac{5}{7}$ and $\frac{56}{77}$ equal to $\frac{8}{11}$, where you plainly ſee that $\frac{8}{11}$ is greater than $\frac{5}{7}$ by $\frac{1}{77}$, becauſe $\frac{56}{77}$ is juſt ſo much greater than $\frac{55}{77}$.

The Sixth Problem.

To multiply one fraction by another.

Make two products, the firſt of the two firſt terms, the ſecond of the ſecond terms of theſe fractions. The firſt product is the firſt term. the ſecond is the ſecond term of another fraction, whoſe quotient is the product deſired. An Example or two will make all this plain.

(1.) *Example.*

To multiply $\frac{4}{3}$ by $\frac{3}{5}$, I make two products, the firſt 12 ariſing from the multiplication of 3 into 4 the numerators of theſe fractions; the ſecond 15 (the two denominators 3 and 5 being multiplied one into another.) The firſt product 12 is the numerator, and the ſecond product 15 the denominator of the fraction $\frac{12}{15}$, whoſe quotient or leaſt terms $\frac{4}{7}$ (found by *Problem* 2.) is the product ſought.

(2.) *Ex-*

(2.) *Example.*

To multiply $\frac{4}{5}$ by $\frac{2}{3}$, the product 8 of 4 by 2 is the first term, and the product 15 of 5 by 3 is the second term of the fraction $\frac{8}{15}$, which (because it cannot be reduced to less terms) is the product sought.

In case the fractions to be multiplyed be improper; *i. e.* such whose numerator exceed their denominator; I would wish the Reader first of all to take out of each of them what they contain equal to a whole number. Secondly, keep the product of the two whole numbers, also the product of the fractions that remained, beside two other products, one made by multiplying the first remaining fraction by the second whole number, the other by multiplying the second fraction by the first whole number. Thirdly gather all these products into one sum, and the business is done.

Example.

To multiply $\frac{17}{3}$ by $\frac{21}{8}$, first I say $\frac{17}{3}$ is equal to $5\frac{2}{3}$ and $\frac{21}{8}$ is equal to $2\frac{5}{8}$. Secondly, I get 10 the product of 5 by 2, also $\frac{5}{12}$ the product of $\frac{2}{3}$ by $\frac{5}{8}$, besides $1\frac{1}{3}$ the

product $\frac{2}{3}$ by 2 or $\frac{2}{1}$, and $3\frac{1}{8}$ the product of $\frac{5}{8}$ by 5. Thirdly, I get the sum of the products found 10, $\frac{5}{12}$, $1\frac{1}{3}$ and $3\frac{1}{8}$ which is $14\frac{7}{8}$, the product sought.

To take any part of a number what you please may be done thus, multiply the number by its part you desire and the work is done. As for example if I wanted $\frac{2}{7}$ of 21, I multiply $\frac{21}{1}$ by $\frac{2}{7}$, so I have $\frac{42}{7}$ which is equal to 6.

The Sixth problem

The Sixth Problem.

To divide one fraction by another.

Make two products, one by multiplying the numerator of the first fraction by the denominator of the second, the other product by multiplying the denominator of the first fraction by the numerator of the second. The first of these products will be the numerator, the second the denominator of another fraction, which reduced (when it may be) to its lowest terms is the quotient sought. An Example or two will make all plain.

Example.

To divide $\frac{4}{5}$ by $\frac{4}{3}$, I make two products, the first 12 by multiplying 4, (numerator of

of the first fraction) by 3 (denominator of the second) and the second 20, by multiplying 5 (the denominator of the first fraction) by 4 (numerator of the second.) The first product is the numerator, the second the denominator of the fraction $\frac{12}{20}$, which being reduced to its lowest terms $\frac{3}{5}$ is the quotient sought.

(2.) *Example.*

To find the quotient of $\frac{8}{15}$ divided by $\frac{2}{3}$, I make two products, the first 24, 8 being multiplyed by 3, and the second 30, 15 being multiplied by 2. The first of these products 24 is the first term and the second product 30 the second term of the fraction $\frac{24}{30}$, whose quotient $\frac{4}{5}$ is the quotient sought.

Whensoever the Reader has occasion to multiply or divide numbers of different kinds, he is to reduce them all to that of the greatest denomination, which is done by dividing the lesser kinds by the number that denotes how often they are contained in the greatest, to which one would reduce them. After which he must find the product or quotient desired by the usual Rules made use of about fractions.

For Example to multiply 6 crowns, 5

ſhillings, 8 pence, 2 farthings, by 8 crowns 1 ſhilling, 4 pence, 3 farthings. Firſt of all I know 5 ſhillings is a crown, 60 pence is a crown, and 240 farthings is a crown. Therefore I divide 5 ſhillings by 5, 8 pence by 60, and 2 farthings by 240, as alſo 1 ſhilling by 5, 4 pence by 60, and 3 farthings by 240, the 3 quotients of the firſt parcel are 1 crown, $\frac{8}{60}$ c. equal to $\frac{2}{15}$ c., $\frac{2}{240}$ c. equal to $\frac{1}{120}$ c. by Rule in page 110, whoſe ſumme by the *4th.Prob.* is $7\frac{17}{120}$: the three quotients of the ſecond parcel is $\frac{1}{5}$ c. $\frac{4}{60}$ c. equal to $\frac{1}{15}$, and $\frac{3}{240}$ equal to $\frac{1}{80}$ whoſe ſum is $8\frac{33}{80}$, and their product is (becauſe I am to multiply them) $59\frac{361}{9600}$, by the *6th.Prob.* The fraction $\frac{361}{9600}$ not admitting of a common diviſor muſt continue as it is.

If one would know the number of which another number is a given part, 'tis but dividing the number by the given part and you have the number deſired. As if one asked of what number is $6\frac{2}{7}$? I divide $\frac{6}{1}$ by $\frac{2}{7}$ and the quotient is $\frac{42}{2}$ equal to 21.

The end of fractions

Of

Of Fractions of Fractions.

JUst as if a whole number be divided into parts, of which ſome only are made choice of, out comes a fraction, whoſe numerator is the number of parts choſen, and its denominator the number of the parts into which the whole is ſuppoſed to be divided: ſo if any particular fraction be divided into ſo many equal parts as it were a whole number, ſome of thoſe may be taken for a numerator of a new fraction, whoſe denominator will be the number of parts into which the firſt fraction was divided. This fraction thus formed is called a fraction of the former fraction, and yet 'tis a part of the whole that was firſt of all divided. Now Reader if thou ſhouldeſt chance to meet with ſuch kind of numbers as theſe, the firſt thing to be done is to reduce them to ordinary fractions of the ſame value, (I mean let the Reader ſo order them, that he may know what part or parts they are of the whole without having any regard to the fractions of which they are fractions) which may be done after this manner. Multiply them one by another, their product is of the

the ſame value with themſelves. For inſtance to reduce $\frac{2}{3}$ of $\frac{4}{5}$ to an ordinary fraction, I multiply one of theſe fractions by the other and the product $\frac{8}{15}$ is equal to $\frac{2}{3}$ of $\frac{4}{5}$. Again to reduce to an ordinary fraction $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$, firſt I multiply the firſt fraction by the ſecond, and the firſt product is $\frac{8}{15}$, ſecondly, I multiply this product $\frac{8}{15}$ by the third fraction $\frac{5}{6}$, and the product $\frac{40}{90}$, or $\frac{4}{9}$, that (by the ſecond *Problem*) is equal to it, has the ſame value $\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{5}{6}$ hath.

The Third BOOK.

Of Rations and Proportions.

HAving ſhewn the Reader the uſual ways of working upon fractions or pieces of numbers, I come in the next place to inſtruct him how to manage Rations or Proportions; the management of whom differs hardly in any thing from the method of handling broken numbers already laid down (rations and fractions being only different names of the ſame thing,) as is fully evident from the writings

tings of those Authors who have Copiously and Learnedly handled the Doctrine of Proportions. 'Tis not unknown to me that Proportions are the principal if not the only Engine by which both ancient and modern Mathematicians, have done those famous exploits, by means of which their names are so deeply engraven in the Chronicle of fame, that time it self (that mighty consumer) will scarce ere be able to eat them out. I mean their Demonstrations, which strike the truly learned with amazement and astonishment, but thunderclap the Sciolist and smatterer. For what are they else each of them, but a long chain of which each link is a proportion; or if you will a long rope made of proportions by which Mathematicians draw Truth out of the dark and deep well of Ignorance (as *Democritus* fancy'd,) Moreover proportions are of that great weight and concernment, that the main if not the whole business of that Angelical Science Algebra is to inform a man how to compute and manage them. Notwithstanding all which I shall not enter into a long discourse about them, my purpose from the beginning being only to lay down the practice of Arithmetick, for should I, I should only turmoil the unexperi-

perienced Readers (for whom this Treatiſe is deſigned) underſtanding , and little or nothing enlighten it. Therefore waving the Theory of Proportions (having firſt of all opened and explained the nature of them) I ſhall fall upon the practical Rules ariſing thence.

In order to the better underſtanding of Proportion I ſhall acquaint the Reader with what ration is, becauſe this gives birth to the other. Now ration is commonly explained thus. 'Tis a mutual relation two numbers bear to each other in reſpect of quantity. For whenſoever the quantity of one number is compared with or ſet againſt the quantity of another, from that compariſon ariſes a certain relation, by which the number ſet againſt the other (which is the ground of this relation) is ſaid to be greater, or leſs, or equal to it ; and in particular to be thus much or ſo much greater or leſs than this ſecond number. The firſt of theſe numbers upon which the relation is founded is called the antecedent, the ſecond to which the relation is referred, the conſequent : and the antecedent being divided by the conſequent, the quotient is called the denominator of the ration. I ſhall not meddle here with the diviſion of ration into that of

of equality and inequality, nor with the many ſubdiviſions of the latter, leſt by this means I ſhould lead the unskilful Reader out of the high road into by paths and preſently loſe him. I ſhall therefore paſs on to Proportions. In treating of which I ſhall ſay nothing of Arithmetical proportion, much leſs of Arithmetical Progreſſion, notwithſtanding each of them contain many excellent proprieties which afford mediums to Arithmeticians, whereby they ſolve abundance of curious and delicate queſtions, all which I ſhall paſs over in ſilence by reaſon they fall not within the compaſs of common practiſe. Confining my ſelf therefore to Geometrical proportion, I ſhall firſt lay down the definition or deſcription of it, next give one diviſion of it, and then fall upon the practical Rules ariſing from it.

Geometrical Proportion is the relation which two equal rations bear to each other. Or if you will, the comparing one ration to another each of which have the ſame denominator. As $\frac{6}{3}$ and $\frac{8}{4}$ are equal rations having the ſame denominator and $\frac{6}{3}$ being equal to $\frac{8}{4}$ is called a Geometrical proportion.

Now every ration requiring two terms, Geometrical proportion that comprehends two rations, muſt neceſſarily re-

quire four. The first of these terms is called the first *antecedent*, the second the first *consequent*, the third the second *antecedent*, the fourth the second *consequent*. And we commonly say as the first *antecedent* to its *consequent*, so the second *antecedent* to its *consequent*. And these proportions are written thus, 6, 3 :: 8, 4; that is 6 is to 3 as 8 to 4. Or the relation between 6 and 3 is equal to that of 8 to 4.

The first and fourth terms of the proportion are called extreams, and the two other the middle terms.

If the two middle terms are equal, the proportion is called continued; and the term that stands for the two middle ones is called a proportional mean.

Geometrical proportion is twofold, either direct or backward.

The direct is when the first term is to the 3d. as the 2d. to the 4th.

The reciprocal or backward is, when the first term of a proportion is to the 3d. as going arseward the 4th. to the 2d.

Now the main business of these proportions is to assist one when three numbers, or three terms of a proportion are given or known to find a fourth unknown.

The principal Rule for this purpose is that commonly called the Rule of Three,

or

or (by reaſon of the great uſefulneſs of it) the Golden Rule.

Now in queſtions falling under this Rule, commonly two terms of the three known are of the ſame kind, to one of which the queſtion is annexed. For inſtance if I ſay 125 Souldiers receive 429 crowns a month, how many crowns at that rate will 10 receive? The two terms 125 Souldiers, and 10 Souldiers are of the ſame kind, and the queſtion hangs to the laſt of them, which is 10. Therefore three terms of a proportion being know one may thus rank them to find the fourth unknown.

You write the term to which the queſtion is annexed in the third place, that of the ſame kind with it in the firſt, the other ſingle term that is of the ſame nature with the fourth unknown in the ſecond place. Theſe terms placed after this manner, if the firſt term is to the ſecond as the third to the fourth unknown, the proportion is direct. But if the firſt term be to the third as the fourth unknown is to the ſecond, the proportion is reciprocal.

For Example, ſhould one ſay that 125 Souldiers receive 429 Crowns a month, after he ſhould ask me how many at that rate ſhould 10 receive, I write 10 Souldiers to

which

which the queſtion is annex'd in the 3d. place, 125 Souldiers that is of the ſame nature in the firſt place, and the ſingle term 429 Crowns of the ſame nature with the fourth unknown in the ſecond place.

Souldiers,	*Crowns :*	*Souldiers,*	*Crowns.*
125 ——	429 ——	—10 ——	—?

Having ſpent a little time in conſidering theſe terms I perceive, the firſt term 125 Souldiers is to the 3d. 10 Souldiers, as the ſecond term 429 is to the fourth unknown. For by how much 125 Souldiers exceed 10 Souldiers, by ſo much muſt the ſecond 429 Crowns exceed the 4th. number unknown. So this proportion is called direct.

The caſe is quite otherwiſe in this queſtion following. 'Tis Recorded in Holy Writ that 153600 Workmen were ſeven years about building the Temple of *Jeruſalem.* How long would 33600 workmen have been about it? I write 33600 workmen, to which the queſtion is annexed in the third place, 153600 workmen of the ſame kind in the firſt place, and ſeven years of a different nature in the ſecond.

Workmen	*Years*	*Workmen.*
153600	7	33600

Af-

Afterward I ſpend a little time in conſidering theſe terms, ſo I find that the firſt term 153600 workmen, is to the third 33600 workmen, as the 4th. term unknown is to the 2d. 7 years. For by how much the firſt term is greater than the third, by ſo much the fourth term unknown ought to be greater than the ſecond. For as much as the number of years ought to be ſo much greater as the number of workmen employed about the Temple is leſs. And hence it comes to paſs that this proportion is reciprocal.

Now the queſtion it ſelf will diſcover to any one heedfully conſidering it, whether the proportion be direct or reciprocal. We uſually judge the proportion to be reciprocal, if the two known terms that are of the ſame kind have a mutual relation, or elſe if they act each of them upon ſomething that is diſtinct from the terms of the proportion. But otherwiſe we look upon the proportion as direct.

Thus in the queſtion of the Temple and the workmen, we judge the proportion is reciprocal, becauſe the two terms 153600 workmen, and 33600 workmen work each upon the Temple they build, which is ſeparate from the workmen and years that make the terms of the proportion.

But

But in the queſtion of the Souldiers and the Crowns we judge the proportion is direct, becauſe the two terms 125 Souldiers, and 10 Souldiers, do not act mutually upon any thing diſtinct and ſeparate from the Souldiers and Crowns that make the terms of the proportion.

As there is a proportion direct and reciprocal, ſo there is the Rule of *Three Direct*, and alſo *Reciprocal.* The direct Rule ſerves to find the 4th. term of a direct proportion, whoſe three firſt terms are known. The reciprocal Rule ſerves to find a fourth term of a reciprocal proportion, whoſe three firſt terms are alſo known. But theſe two Rules may be brought into one, *i. e.* a *right.* For if you know the proportion is reciprocal 'tis but placing the firſt term in the third place, and the third term in the firſt, and you convert the proportion into a direct one. Thus in the queſtion of the Temple and workmen which make the proportion reciprocal.

Workmen.	*Years.*	*Workmen.*
153600	7	33600

You convert this proportion into a direct by writing 33600 7 153600

Of

Of the Rule of Three Direct.

The first Problem.

THree terms of a direct proportion being known to find out the fourth.

First you write the term to which the question is annexed in the third place, that of the same kind in the first, and the other of a different nature in the second.

Secondly, These terms being thus placed, you multiply the second term by the third, and divide their product by the first, the quotient found is the 4th. term of the proportion.

The first Question.

125 Souldiers receive 425 Crowns a month, how many Crowns at that rate will 10 receive?

First 125 Souldiers and 10 Souldiers do not mutually act upon any thing distinct from the terms of the proportion, so the proportion is direct; therefore I write 10 Souldiers, to which the question is annexed

nexed in the 3d. place, 125 Souldiers that is of the ſame kind in the firſt, and 425 Crowns in the ſecond. Secondly, theſe three terms being thus placed, I multiply the ſecond term 425 by the third 10, and the product 4250 I divide by the firſt term 125, the quotient of this diviſion is 34, and this quotient is alſo the 4th. term ſought. Therefore 10 Souldiers will have 34 Crowns a month.

Souldiers	*Crowns*	:	*Souldiers*	*Crowns.*
125 ——	-425	: :	10 ——	34

The Second Queſtion.

A high Tower makes a ſhadow of 164 foot, and a pole 26 foot long, ſtanding bolt upright caſts a ſhadow of 5 foot, the quere is how high the Tower is?

Firſt five foot of ſhadow, and 164 foot of ſhadow do not act mutually upon any thing ſeparate from the terms of the proportion; ſo the proportion is direct: Therefore I write 164 feet of ſhadow the Tower caſts, to which the queſtion is annexed in the 3d. place, the five foot of ſhadow the pole caſts in the firſt, and 26 feet the length of the pole in the ſecond. Secondly, Theſe three terms being thus placed

ced I multiply 26 by 164, and their product I divide by 5, the quotient thence arising is $852\frac{4}{5}$, and this is that was ſought for. So that the height of the Tower is 852 feet, 9 inches and almoſt two Barly-corns, the fraction $\frac{4}{5}$ being reduced by the Rule laid down page 49.

Feet of the ſhadow the Pole caſts.	*Feet of the height of the Pole.*	*Feet of the ſhadow the Tower caſts.*	*Feet of the height of the Tower.*
5 ——	—26	164 ——	. $852\frac{4}{5}$

The third Queſtion.

A Traveller is to go a journey of 138 miles, 16 of which he goes in 3 dayes, you ask after that rate in how long time he can perform that journey. I write 138 to which the queſtion hangs in the third place, 16 of the ſame kind with it in the firſt place, and 3 days differing from the other two in the ſecond place, which being placed according to due order, I multiply 138 by 3, and the product 414 I divide by 16, the quotient $25\frac{14}{16}$ is the 4th. proportional ſought; and thus ſtands the

miles. dayes : miles. days.

work, 16 — 3 138 — $25\frac{14}{16}$ or $25\frac{7}{8}$, which fraction reduced by Rule in pag. 49. to hours

hours is 21 hours, ſo that the perſon goes his journey in 25 dayes and 21 hours.

The Fourth Queſtion.

How much coſts 1 pound, if 64 pound of Spice coſt 84 ſhillings, thus ſtands the work, $\begin{matrix} \textit{lib.} & \textit{ſhill.} & : & \textit{lib.} \\ 100 & 84 & & 1 \end{matrix}$ the number ſought is $\frac{84}{100}$, which fraction (by Rule in pag. 49.) reduced to groats is $2\frac{52}{100}$, which laſt fraction again reduced to pence is $2\frac{8}{100}$, ſo that each pound will coſt 10 pence, and a ſmall matter more.

Now when moneys of a different value, and different weights come to be workt upon, you muſt reduce the greater ones to the denomination of the leaſt.

Examples of mixt numbers.

HOw much does 1 pound of Pepper coſt, if 120 pound, 14 ounces are bought at 94 ſhillings, 9 pence. In this Queſtion the firſt and ſecond terms conſiſt of mixt numbers, therefore the greater are to be reduced to the leaſt.

Be-

Becauſe 16 ounces make a pound, I multiply 120 pound by 16, to the product 1920 I add 14 ounces, the ſum is 1934 ounces. After the ſame manner I multiply 94 by 12, and to the product 1128 I add 9 the ſum is 1137 pence.

Therefore I ſay 1934 ounces are bought at 1137 pence, what may one buy 1 pound or 16 ounces for, your work ſtands thus,

ounces. *pence.* *pound.*

1934 1137 1 makes $8\frac{272}{1934}$, the fraction being reduced to farthings, 1 pound will coſt 8 pence half-peny farthing, and ſomewhat above.

A Merchant buys 24 pound, 6 ounces at 24 ſhillings, 6 pence, 1 farthing, for what may one have a 1000 pound?

Firſt 24 pound, 6 ounces reduced to ounces make 390 ounces (ſuppoſing 16 ounces to a pound) for the firſt term, then reduce 1000 by multiplying of it by 16 to 16000 ounces, which let be the third term. Now 24 ſhillings, 6 pence, 1 farthing, make together 1177 farthings, for the ſecond term. Say therefore 390 ounces are bought with 1177 farthings, and conſequently 16000 ounces will coſt $48287\frac{7}{39}$, or $\frac{1883200}{39}$ farthings.

ounces. *farthings.* *ounces.* *farthings.*

390 1177 :: 1600 $48287\frac{7}{39}$,

which

which divided by 48, (according to Rule in pag. 122) the number of farthings that are in a ſhilling, makes $1005\frac{1840}{1872}$ ſhillings or fifty pound five ſhillings, the laſt fraction reduced to pence, is $11\frac{1488}{1872}$, and this ſame to farthings is $3\frac{1336}{1872}$. So that the laſt proportional number is 1005 ſhillings or 50 pound 5 ſhillings, 11 pence, half-peny farthing and ſomewhat more.

Examples of Broken Numbers.

HOw much will $\frac{7}{8}$ of 1 pound of Pepper coſt, if a man buy $\frac{1}{2}$ a pound for $\frac{3}{4}$ of a ſhilling. The work ſtands thus $\frac{1}{2}$ —— $\frac{3}{4}$ —— $\frac{7}{8}$.

I multiply (by Rule in pag. 120) $\frac{3}{4}$ by $\frac{7}{8}$, and the product $\frac{21}{32}$ I divide (by Rule in pa. 122) by $\frac{1}{2}$, the quotient is $\frac{42}{32}$ or $1\frac{10}{32}$, *i. e.* $1\frac{2}{16}$, by Rule in page 110.

32 yards of Cloth are bought at $22\frac{1}{2}$ ſhillings, how much will $8\frac{1}{2}$ coſt? By Rule in pag. 113. I reduce $22\frac{1}{2}$ to this fraction $\frac{45}{2}$ & $8\frac{1}{2}$ to $\frac{17}{2}$, then I multiply $\frac{45}{2}$ by $\frac{17}{2}$ by Rule in page 120, and the product $\frac{765}{4}$ I divide (by Rule in pag. 122) by $\frac{32}{1}$, the quotient is $\frac{765}{128}$ or $5\frac{125}{128}$ ſhillings, the fraction adhering to 5 being

being reduced to pence and farthings, the fourth number in the proportion is 5 shillings 11 pence, and almost 3 farthings.

Of the Backward or Reciprocal Rule of Three.

The second Problem.

THree terms of a Reciprocal proportion being known to find out a fourth.

The rise and foundation of this Rule is a certain identity, sameness or equality, which though now and then may lay hid in the Question, yet to the heedful Practitioner 'twill easily appear.

To perform this Rule, first you must write the term to which the Question is annexed in the third place, that of the same kind in the first, and the other of a different kind in the second.

Secondly, These terms thus placed, you are to multiply the first by the second, and divide their product by the 3d. the quotient

ent thus found is the fourth proportional term.

(1.) *Example.*

If 153600 workmen built the Temple of *Hierusalem* in seven years, in how many years would 33600 workmen have built it?

First 153600 workmen and 33600 workmen act mutually upon the same Temple, for the first company of workmen built it in a term of time so much the shorter as they were more in number, on the other hand the last gang built it in a term of time so much the longer as they were fewer in number. And by this means the proportion is reciprocal. Therefore I write 33600 workmen to which the Question is annexed in the third place, 153600 workmen in the first, and 7 years in the second. Secondly I multiply 153600 by 7, and the product I divide by the 3d. 33600, the quotient is 32, and this quotient is the 4th. term sought: the 33600 workmen therefore would have built it in 32 years.

Workmen.	*Years* :	*Workmen.*	*Years.*
153600	7 : :	33600	32

(2d.) *Ex-*

(2.) *Example.*

If *Hierusalem* besieged by the Army of *Vespatian* had Provisions enough to maintain 2000000 men 10 years, how long would it have maintained 400000?

First 2000000 and 400000 have a mutual regard to the provisions, for they may be sustained so much the longer by them as they are fewer in number, and so much the shorter space as they are more in number. Upon which account the proportion is apparently reciprocal, so I write 400000 men to which the Question is annexed in the third place, 2000000 men in the first, and 10 years in the second, secondly I multiply the first term 2000000 by the second 10, and I divide the product by the third 400000, the quotient is 50, which is the fourth proportional term sought. So that *Hierusalem* could have maintained the 400000 men 50 years.

Men	*Years* :	*Men*	*Years*
2000000 —	7	400000 —	50

(3d.) *Ex-*

(3.) *Example.*

If you muſt have 12 yards of Stuff, whoſe breadth is $\frac{3}{4}$, to make a Garment; how many yards of Stuff muſt you have whoſe breadth is but $\frac{2}{3}$?

Firſt you ſee plainly the proportion is reciprocal, becauſe the two breadths of the Stuffs bear a mutual relation to the Garment, for the wider the Stuff, the fewer yards you muſt have, and on the contrary the narrower the Stuff the more yards you muſt have. The proportion is therefore backward, and I write the breadth $\frac{2}{3}$ to which the queſtion hangs in the 3d. place, the breadth $\frac{3}{4}$ in the firſt, and the 12 yards in the ſecond. Secondly I multiply the firſt term $\frac{3}{4}$ by the ſecond 12, and the product 9 I divide by the third $\frac{2}{3}$, the quotient is $13\frac{1}{2}$. And this quotient is the number I ſought after. So that you muſt have 13 yards and half of Stuff $\frac{2}{3}$ wide to make a Garment, with the proviſo you ſhould have had 12 yards of Stuff $\frac{3}{4}$ wide.

firſt breadth.	*yards.*	*ſecond breadth,*	*yards.*
$\frac{3}{4}$	12	$\frac{2}{3}$	$13\frac{1}{2}$

Ano-

Another way to perform the Rule of Three Reciprocal.

IF you perceive that the queſtion propoſed belongs to reciprocal proportion, and you ſo order its three known terms, that the term to which the queſtion is annexed may be in the firſt place, that of the ſame kind in the third, the other ſingle one and of a different nature in the ſecond, the queſtion may be ſolved by multiplying the ſecond term by the third, and dividing the product by the firſt. For the quotient of ſuch a kind of diviſion will be the term unknown that was ſought.

For Example knowing that the queſtion of the Workmen that built the Temple, belongs to reciprocal proportion, if you write the 33600 workmen to which the queſtion is annexed in the firſt place, and 153600 workmen of the ſame kind in the third, and the ſeven years in the ſecond, you will ſolve the queſtion by multiplying the ſecond term 7 by the third 153600, and dividing the product by the firſt term

33600, for the quotient of this division which is 32, will be the number unknown that was desired.

Workmen.	*Years.*		*Workmen.*	*Years.*
33600	7	: :	153600	32

Another way of performing the Rule of Three both Direct and Reciprocal.

WHen the proportion is direct, you may very often more readily do your business by dividing the second term by the first, and multiplying the quotient by the third; Or which is all one, if you think it easier, by dividing the third term by the first and multiplying the quotient by the second, for the product found after either of these ways will be the fourth term of the proportion. That the use of this may appear I shall apply it to the first Example of the preceding Rule.

The three known terms 125 Souldiers, 425 crowns and 10 Souldiers being placed after

after the uſual manner, I divide the ſecond term by the firſt, and I multiply the quotient $\frac{425}{125}$ by the third 10, and the product is 34. And this number of Crowns ought 10 Souldiers to receive.

To ſolve the queſtion of *Hieruſalem* and her Citizens beſieged. The three terms known 400000 men, 10 years and 2000000 men being ſo ordered that they may be the three firſt terms of a direct proportion. I divide the third term 2000000 by the firſt 400000 (which is done by dividing only 20 by 4) and I multiply the quotient 5 by 10 and the product is 50, and this number is that of the years the City could have maintained 400000 men.

Men.	*Years.*		*Men.*	*Years.*
400000	10	: :	2000000	50

If in ſolving this Queſtion I had divided the ſecond term by the firſt, and multiplied the quotient by the third, it had been all one, for this way alſo I ſhould have attained my purpoſe.

The Reader may take notice that the pains taken about the Rule of Three may oftentimes be very much eaſed thus. If

of three numbers given and orderly placed the firſt and ſecond, or firſt and third admit of a common diviſor, dividing them by their common diviſor, let the quotients thence ariſing ſtand in their ſtead and then go on with the work. If the numbers chance to be even, halve them till you light upon an uneven. After that if they admit of a common diviſor divide them by it. By this means the work will be very much eaſed.

As 4 pound are bought with 42 ſhillings, what will 85 coſt?

Becauſe 4 and 32 are compound numbers, being divided by their common diviſor 4, put the quotients 1 and 8 in their room thus, 1—8 : : 85, multiply 85 by 8, the product is 680.

42 yards of Cloth are bought with 27 ſhillings, for what may you have 56?

Reduce 42 and 56 by halving them to 21 and 28, which again divided by their common diviſor 7, inſtead of them you have 3 and 4. Moreover the firſt term 3 and the ſecond 27 being divided by their common diviſor 3, you have in their room 1 and 9. Therefore the example being thus reduced to its leaſt terms will be 1———9———4, multiply 9 by 4, the product will be 36; view the ſcheme.

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$$\begin{array}{lcl} 42 \text{ ------ } & 27 & \text{ ------ } 56 \\ 21 & & 28 \\ 3 & & 4 \\ \hline 1 & 9 & 4 \text{ makes } 36 \end{array}$$

The ſame courſe may be taken about the denominators of fractions. For if the denominator of the 1ſt. and 2d. fraction or of the firſt and third admit of a common diviſor, divide each of them by it. For inſtance if $\frac{1}{2}$ gives $\frac{3}{4}$, what will $\frac{7}{8}$ give? Divide the denominator of the firſt and ſecond fraction by 2, ſo they are reduced to 1 and 2. The Example will ſtand thus, $\frac{1}{1}$ —— $\frac{3}{2}$ —— $\frac{7}{8}$. the 4th. number is $\frac{21}{16}$ or $1\frac{5}{16}$.

When the numbers are mixt, that is, whole numbers with fractions adhering to them, firſt of all reduce them to fractions by Rule in page 113. As $4\frac{3}{8}$ gives $25\frac{1}{2}$, what will $24\frac{3}{4}$ give?

This Example reduced to fractions will ſtand thus, $\frac{35}{8}$ —— $\frac{51}{2}$ —— $\frac{99}{4}$. Afterward reduce the denominator of the firſt and third fraction 8 and 4 to 2 and 1; the Example after this alteration ſtands thus, $\frac{35}{2}$ —— $\frac{51}{2}$ —— $\frac{99}{1}$. Again Reduce

the denominators of the first and second to 1 and 1. The numbers after this manner reduced to their least terms stand thus, $\frac{3}{1}$ give $\frac{5}{1}$, what will $\frac{9}{1}$ give; The work being performed according to the accustomed manner you have for the 4th. proportional this number $144\frac{9}{35}$.

The Proof of the Rule of Three.

To be assured the fourth term found is the right one when the proportion is direct, you need do no more but get the products of the extreams and middle terms, if these two products are equal, your work is rightly performed, if unequal, 'tis faulty.

But when the proportion is Reciprocal, you are to get the product of the two first terms, and that of the two last, If these two products are equal, the business is truly done, if unequal the contrary, and you must begin your work afresh.

Of

Of the Rule of Three Compound.

THis Rule teaches to find a number which is to another compounded of ſeveral others, as a third is to a fourth ; compounded alſo of ſeveral others. For Example the carriage of 200 weight of goods brought 300 miles, comes to 4 Crowns ; how much will the carriage of 400 weight brought 500 miles come to?

'Tis apparent in this queſtion that you require a price which may not only be proportional to the weight of the goods which you bring, but alſo to the length of the way you bring them.

In this queſtion and ſuch like you bring into play more than three terms, but there are but three principal ones, upon which the other depend. Thus in our queſtion there are five known terms propoſed, 200 weight of goods, 300 miles, 4 Crowns, 400 weight of goods and 500 miles, of all which the three principal are 200 weight of goods to which 300 miles is related, 4 Cr.

and 400 weight of goods upon which depends 500 miles. Now if the two principal terms of the queſtion being of the ſame kind, and to whom the queſtion is annexed, act each of them upon one of the terms that depends on them the proportion is Reciprocal, if otherwiſe 'tis direct.

For Example if I ſay 4 Labourers manure 8 Acres of Ground in 3 days, in how long time will 3 Labourers manure 24 Acres?

The two principal terms and of the ſame kind are the 4 and the 3 Labourers. Now becauſe the 4 Labour. work upon the 8 Acres, which is a term depending thereupon, and the 3 other upon the 24 Acres which depend alſo upon it; I know the queſtion belongs to the Compound and Reciprocal Rule of Three.

Of the Direct Rule.

The third Problem.

ALL the terms ſave one of a compound and direct proportion being given, to find this ſame unknown term.

Firſt you write the principal term to which the queſtion is annexed in the third place, and under it all thoſe that depend on it, the other chief and of the ſame kind in the firſt, and under it thoſe that bear relation to it, the other ſingle term and of a different nature in the ſecond.

Secondly, you write under each term of the proportion the compound number. This reduces all the terms given to the three firſt terms of a direct proportion, whoſe fourth term is that deſired.

(1.) Example.

The carriage of 200 weight of goods brought 300 miles comes to 4 crowns, how much will the carriage of 400 weight brought 500 miles come to?

First I write the 400 weight of Goods which is the chief term to which the question is annexed in the third place of a proportion, and under it the 500 miles depending upon it, then I write the 200 weight of goods, which is the other chief term of the same kind in the first place, and under it the 300 miles depending thereon, and the 4 Crowns that is a single term and differing from the first in the second place. Secondly, I write 60000 that is the product of 200 weight by 300 miles under the first place, the solitary term 4 under the second, and 200000, *viz.* the product of 400 weight by 500 miles under the third place. This reduces the five terms given to the three first of a direct proportion, whose fourth term $13\frac{1}{3}$ denotes how much the carriage of 400 weight will come to. And thus the 400 weight will come to 13 Crowns, and 5 Groats, being brought 500 miles, supposing as we do here, that 200 weight brought 300 miles comes to 4 Crowns.

200 weight		400 weight	
	4 Crowns : :		
300 miles		500 miles	?
60000	4 Crowns : :	200000	$13\frac{1}{3}$ cro.

(2d.) *Ex-*

(2.) *Example.*

8 Merchants that have a 100 Buts of Wine a piece at 15 Crowns a But, get amongst them 3200 Crowns, how many crowns will a 11 Merchants that have each 128 Buts of Wine at 12 crowns a But gain amongst them?

First perceiving as in the former Example that the proportion is direct, I write 11 Merchants, which are the chief term to which the question is annexed in the third place of a proportion, and under it 128 Butts of Wine, and also the 12 crowns, the price of each But, for these two terms depend upon it. I write the 8 Merchants, which are the other chief term and of the same kind with the 11 Merchants in the first place, and under it the 100 Butts of Wine that belonged to each Merchant, and also the 15 Crowns the price each Butt was valued at, for these two terms are as it were appendages of the 8 Merchants. And lastly I write the 3200 crowns, *viz.* the single term and of a different nature in the second: secondly, I write under the first 12800, *i. e.* the solid number made by multiplying the two appendages

100

100 and 16 interchangeably into the principal number 8, I write the 3200 crowns under the 2d. under the third 16916, *viz.* a ſolid number made by the mutual multiplication of 11, 128 and 12 into one another. This reduces all the terms known to the three firſt terms of a direct proportion, and 4204 which is the laſt term of this proportion will be the number of crowns the 11 Merchants ought to get.

8 *Merchants* 100 *Butts* 16 *Crowns*	3200 *Crowns* ::	11 *Merchants* 128 *Butts* 12 *Crowns*	?
12800	3200 *Crowns* ::	16896	4224 *Crowns.*

The fourth Problem.

All the terms of a compound and reciprocal proportion being known ſave one, to find this unknown term.

Firſt you write the principal term to which the queſtion is annexed in the firſt place, and under it the appendages of the other principal term that is of the ſame kind; and you write going arſeward this other term in the third place, and under it all thoſe terms that have a relyance upon that to which the queſtion hangs; and the ſolita-

ſolitary term which is of a different nature in the ſecond.

Secondly, you write under each place of the proportion, the product of all the numbers that are there. This reduces all the terms given to the three firſt terms of a direct proportion, whoſe fourth is the number deſired.

(1.) *Example.*

4 Labourers manure 8 Acres of Land in 3 dayes, in how many dayes will 3 Labourers manure 24 Acres?

Firſt perceiving the compound proportion to be Reciprocal, I write 3 Labourers, which are the chief term to which the queſtion is annexed in the firſt place of a direct proportion, and under it the 8 Acres which is an appendage of the 4 other Labourers. And I write reciprocally 4 Labourers which are the other chief term, and of the ſame kind in the third, and under it the 24 Acres, which the 3 Labourers manur'd. And the 3 days which are the ſolitary term, and of a different nature in the ſecond, Secondly, I write 24 the product of 3 Labourers by 8 Acres which the 4 other manure under the firſt place,

place, the 3 dayes under the ſecond, and 96 the product of 4 Labourers by the 24 Acres, that the 3 other manure, under the third place. This reduces the five terms given to the 3 firſt of a direct proportion, whoſe fourth term 12 denotes the number of dayes during which the 3 Labourers will manure 24 Acres of Land, with the proviſo (as is here ſuppoſed) that 4 Labourers manure 8 Acres in the ſpace of 3 dayes.

3 *Labourers*		4 *Labourers*	
	3 *dayes* : :		
8 *Acres*		24 *Acres*	?
24	3 *dayes* : :	96	12

(2.) *Example.*

If a 100 men drink up 12 veſſels of Wine containing 500 pints a piece in a month, which makes 30 days, in how many days will 1240 men drink up 64 Veſſels of Wine holding 600 pints each?

First perceiving that the proportion muſt needs be Reciprocal, I write 1240 men to which the queſtion hangs in the firſt place of a direct proportion, and under

under the 12 Veſſels that the other 100 men drank, and under it alſo the 500 pints each of them hold. And reciprocally I write the 100 men in the third place, and under the 64 veſſels of Wine that the 1240 men drank, and alſo under it the 600 pints each of them contains. And in the ſecond the term 30 dayes differing from the former. Secondly, I write 7440000 or the ſolid number made by the multiplying the 3 terms 1240, 12 and 500 one into another under the firſt place. The 30 days under the ſecond, and 3840000 or the ſolid number made by the multiplying the three terms, 100, 64 and 600 one into another under the third place. This reduces all the terms known to the three firſt of a direct proportion, whoſe fourth term $15\frac{15}{31}$ denotes the number of days during which the 1240 men will drink up their 64 veſſels holding 600 pints a piece.

1240 *Men*		100 *Men*	
12 *Veſſels*	30 *days* : :	64 *Veſſels*	
500 *Pints*		600 *Pints*	?
7440000	30 *dayes* : :	3840000	$15\frac{15}{31}$ *dayes.*

Of the Rule of Fellowship.

THere are ſeveral Rules that ſpring from the Rule of Three, as it were ſuckers from that Root.

I ſhall here give the Reader an account of the chiefeſt of them, which are the Rule of Fellowſhip, Rules of Alligation, and thoſe of Falſe Poſition.

The Rule of Fellowſhip is that that teaches to divide a number given into ſeveral parts which ſhall be proportional to other aſſigned numbers. It borrows its name from the Companies uſually eſtabliſhed amongſt Merchants to Traffique together, because in this caſe its uſe is of abſolute neceſſity.

The fourth Problem.

To divide any number given into ſeveral parts that are proportional to other numbers likewiſe known.

Firſt you write the ſum of theſe known numbers in the firſt place of a direct proportion

portion, the other number that is to be divided in the ſecond, and each of the aſſigned or given numbers in the third.

Secondly, you make uſe of the Rule of Three as often as there are numbers in the third place, and you find ſo many new terms, which are the proportional parts ſought.

(1.) *Example.*

Three Merchants make a joint-ſtock of 10000 Crowns, with which they get 4000. The firſt Merchant put into the common ſtock 2000 Crowns, the ſecond 5000, and the third 3000, you ask what each Merchant ought to receive of the whole gain, *viz.* the 4000 crowns that ſhall be proportionable to what he put in?

Firſt I write 10000 the ſum total of the moneys put into the ſtock, in the firſt place of a direct proportion, 4000, the number of crowns that makes the whole gain, of which each Merchant has his ſhare according to what he put in, in the ſecond place: and the three numbers 2000, 5000 and 3000, to wit the crowns each Merchant put into the ſtock in the third place. Secondly, I make uſe of the Rule of three thrice

thrice, and I find the three fourth terms, 800, 2000 and 1200, which are those that from the beginning were desired.

Therefore the first Merchant ought to receive 800 crowns of the whole gain, the second 2000, and the third 1200.

The whole contribution.	*The whole gain.*		*Particular contributions*	*Particular gains.*
10000	4000	: :	2000 5000 3000	800 2000 1200

(2.) *Example.*

Four Merchants Trading upon a joynt-stock of 16000 crowns, lose 4000. The first Merchant put into the common stock 2000 crowns, the second put into it 5000, the third 3000 and the fourth 6000, the question is what part of the loss each Merchant ought to bear, as shall be proportionable to the sum he put in?

First I write the whole contribution 16000 in the first place of a direct proportion, the whole loss 4000 crowns in the second, and each particular contribution in the third. Secondly, I make use of the direct

direct Rule of Three four times, and I find the four terms 500, 1250, 750 and 1500, which are those I sought after.

Therefore the first Merchant must come off loser 500 crowns, the second 1250, the third 750, and the fourth 1500. And if you substract each mans loss from the sum he at first put in, the first that put in 2000, will take out but 1500, the second that put in 5000, will take out but 3750, the third that put in 3000 will take out but 2250, lastly the fourth that put in 6000 will take out but 4500.

The total contributions	*The total loss*	*Particular contributions*	*Particular losses*	*Particular remainders.*
16000	4000 ::	2000	500	1500
		5000	1250	3750
		3000	750	2250
		6000	1500	4500

Of

Of the Compound Rule of Fellowship.

The fifth Problem.

THis Rule teaches to divide any given number into several parts, so that they may be proportional to divers other compound numbers that are also given or known.

For Example four Merchants Trade upon a joynt-stock, toward which the first contributed 20 Crowns which lay in 4 months, the second 40 Crowns which was in 5 months, the third 60, that continued in the bank 6 months, and the fourth 80 that continued 7 months, and they gain 240 Crowns. The question is what each Merchants gain is with regard to the sum he contributed, and the time it lay in the common-stock?

'Tis evident that in order to the solving of the question you must make use of the compound numbers of the mony each Merchant put in and of the time it continued

nued in the joynt-ſtock, *viz.* the product of each ſum multiplyed by the number denoting the ſpace of time it lay in. Having theſe ſeveral products you take their ſum which is 1200. You write this ſum in the firſt place of a proportion, the whole gain 240 in the ſecond, and each of the compound numbers or products in the 3d, the 4th. term of the 1ſt. proportion allows the firſt Merchant 14 Crowns for his ſhare of the gain, the fourth term of the ſecond 48 the ſecond Merchant gets, the fourth term of the third 72 which is the gain of the third Merchant, the fourth term of the laſt 12 for the gain of the fourth.

1ſt. 20 *Crowns for* 4 *months.*	*2d.* 40 *Crowns for* 5 *months.*	*3d.* 60 *Crowns for* 6 *months.*	*4th.* 80 *Cr. for* 7 *mon.*
Products 80	200	360	560 *their ſum* 1200.

Sum of the products.	*whole gain*	*products*	*particular gains.*
		80	16 *gain of the firſt.*
1200	240 : :	200	40 *the ſeconds gain.*
		360	72 *the thirds gain.*
		560	112 *the fourths gain*

The

The Rule of Alligation.

THis Rule teaches to mix things of different values, ſo that their mixture may make a compound, that has a ſet value between the values of the other two.

For Example a Goldſmith has two ſorts of Gold, one is worth 30 ſhillings the oun. the other ſort is worth but 24, you ask what mixture muſt be made of theſe two ſorts of Gold, that one may have an ounce whoſe value is 28 ſhillings? The prices 30 and 24 ſhillings are the prices of two ſorts which you are to run into one, one ounce is the compound or maſs this mixture ought to make, and 28 is the ſet price which falls between 30 and 24.

The Sixth Problem.

To unite two things of different values, ſo that the compound thence ariſing may have a middle value between them.

Firſt get the difference between the greater price and the middle one, and the diffe-

difference between the middle price and the least. Afterward you make two ranks of direct proportion writing the sum total of the differences for the first term, the thing whose value is the middle price for the second, and each of the differences for the third.

Secondly, you seek for the fourth term of each proportion, and these two terms denote alternately, the second what you ought to take of the thing that carries the greater price, and the first, what you must take of the thing that bears the less price, so that you may come by the mixture you desire. The Examples will make all this plain.

(1.) *Example.*

A Goldsmith has two sorts of Gold, one is worth 30 shillings the ounce, and the other is worth but 24 shillings the ounce, what mixture must be made of these two sorts, so that you may have an ounce worth 28 shillings?

First the difference between the greater price 30, and the middle one 28 is 2, and the difference between 28 the middle price and 24 the lesser is 4, therefore I write 6 the

the ſum of the differences in the firſt place of two direct proportions, 1 ounce or the thing that carries the middle price in the ſecond, and each of the differences 2 and 4 in the third. Secondly, I ſeek for the fourth term of each proportion. Theſe terms are $\frac{1}{3}$ and $\frac{2}{3}$ whoſe ſum is 1, and the term $\frac{1}{3}$ denotes what you muſt take of the Gold of the leaſt value, and the term $\frac{2}{3}$ alternately what you muſt take of the Gold of the greateſt price, ſo that you may have 1 ounce whoſe value is the middle rate. And verily the third you are to take of 24 is juſt 8, and the two thirds you are to take of 30 ſhillings comes to 20, ſo that theſe two ſums added together make the middle rate 28.

greateſt price,	*middle price,*	*leaſt price,*	*1ſt. differ.*	*2d.differ.*
30 *ſhill.*	28	24	2	4

Sum of the differ.	*The compound*	*differ.*	*Parts of the mixture alternately taken.*
6	1 : :	2	$\frac{1}{3}$ *of an oun. of* 24 *s.*
		4	$\frac{2}{3}$ *of an oun. of* 30 *s.*

(2.) *Example.*

A Vintner has two ſorts of Wine, one worth 5 pence a pint, the other 2 pence, you

you ask what I must take of each to fill a vessel that holds 400 pints, so that it shall be worth 5 pound.

First of all I must find out the value of a pint of that Wine that is to fill the cask, which I know thus, if 400 pints be worth 5 pound what will a pint come to? the fourth term is the value of 1 pint which is $\frac{1}{80}$, this fraction multiplyed first by 20 to reduce it to shillings, and after that by 12 to reduce it to pence stands thus, $\frac{240}{80}$ equal to 3. This done I say if two sorts of Wine be worth, one 5 pence a pint and the other 2 pence, what mixture must I make that I may have 400 pints, that each pint may be worth 3 pence, and I dispatch the question after this manner.

First, the difference between the greatest price 5, and the middle one 3 is 2, and the difference between 3 and 2 the least price is 1, therefore I write 3 the sum of the differences for the first term, 400 pints (the value of each of which is 3 the middle rate) for the second term, and each of the differences 2 and 1 for the two third terms. Secondly I find out the 4th. term of both these proportions. These terms are $266\frac{2}{3}$ and $133\frac{1}{3}$, whose sum is 400. And the first of these terms $266\frac{2}{3}$ tells me how ma-

ny pints I must take of the Wine that is of the least value, and the other $133\frac{1}{3}$ how many pints I must take of the Wine that is of the greatest value, so that I may have 400 pints worth 400 times the middle price 3 pence *i. e.* 1200 pence, that is 100 shill. or 5 pound.

400 pints. 5 pound : : 1 pint $\frac{1}{80}$ of a p. which comes to 4 p.

greatest price.	*mid. pr.*	*least pr.*	*first differ.*	*second differ.*
5 pence	*3 pence*	*2 pence*	2	1

sum of dif.	*middle allay.*		*differ.*	*Particular and Alternate mixtures.*
3	1	: :	2	$266\frac{2}{3}$ *pints of 2 pen. a pint*
			1	$133\frac{1}{3}$ *pints of 5 pen. a pint*

The seventh Problem.

To Alligate or tye one to another things of different rates, so that they may make a compound of a middle rate.

First you make of all the prices two parcels, from each of which you take the sum of their differences, and you multiply alternately, the first sum by the number of prices that helped to find the second ; and the second sum, by the number of prices that helped to find the first. Afterward you

you frame two ranks of direct proportions writing the sum of the two products found in the first place, that that has the middle allay in the second, and each of the two products in the third.

Secondly, you seek for the two fourth terms of the proportions, and these terms direct alternately, the first, what you must take of the things whose rates helped to find the second sum, and the second, what you ought to take of the other things that remain, and whose prices also helped to find the first sum.

(1.) *Example.*

If an ounce of Saffron is worth 100 pence, an ounce of Cinnamon 8 pence, of Nutmegs 6 pence, of Pepper 4 pence, and of Cloves 3 pence. What must you take of each to make a mixture of 21 pound, which shall be worth 9 shillings and 4 pence a pound.

First of all if 1 pound which makes 16 ounces be worth 9 and 4 pence which makes 112 pence, 1 ounce which is $\frac{1}{16}$ part of a pound will be worth $\frac{1}{16}$ part of 112 pence that is, $\frac{112}{16}$, or 7 pence. And if 1 pound make 16 ounces, 21 pound will make 21 times 16 or 336 ounces.

ounces.	*pence*	: :	*ounces.*	*pence*	*pound.*	*ounces*	: :	*pou.*	*oun.*
16	112	: :	1	7	1	16	: :	21	336

This done I ſay, if 1 ounce of Saffron be worth 10 pence, an ounce of Cinnamon 8 &c. what muſt I take of each ſort to have a mixture of 336 ounces worth 7 pence an ounce? And I diſpatch the queſtion after this manner.

Firſt the ſum of 3 and 1, which are the differences between the two greateſt rates 10 and 8, and the middle rate 7 is 4; and the ſum of 1, 3, and 4 which are the differences between the middle rate 7 and the leſſer rates 6, 4 and 3 is 8. Therefore I multiply alternately the firſt ſum 4 by 3 the number of the prices 6, 4 and 3, which help'd to find the ſecond ſum 8, and I multiply this ſecond ſum 8 by 2 the number of prices 10 and 8, which aſſiſted to find the firſt ſum 4. Afterward I make two ranks of direct proportions, writing the ſum of the products in the firſt place, the 336 ounces that have the middle allay in the ſecond, and the two products 12 and 16 in the third. Secondly, I find out the two fourth terms of the proportions. Theſe terms are 144 and 192, and their ſum is equal to

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336 the number of the ounces which were to make the compound. And the first of these terms 144 shews you what you must take of the mixture of the Pepper, Nutmegs and Cloves, whose prices helped to find the second sum, and the other term 192 directs what you must take of the mixture of the Saffron and Cinnamon, whose prices help'd to find the first sum. And to determine yet more particularly what you must take of each sort, you may divide 144 into three equal parts, and 48 the third of 144 will be the number of ounces of each of the three second sorts, and likewise 96 the half of 192, will be the number of ounces of each of the first sorts.

A Scheme of the Work.

	prices	*their differ.*	*and their*	*sum?*
Saffron	10 *pence*	3		1
Cinnamon	8	1		3
middle rate	7	*first sum* 4		4
Pepper	6	*by* 3	2*d. sum*	8
Nutmegs	4	1*st.prod.* 12	*by*	2
Cloves	3	2*d.prod.* 16	2*d. product*	16
	sum of the products	28		

Sum of the products	*comp. of mid. allay.*	*products*	*parts of the mixture alternately taken.*	
28	336 : :	12	144	48 *oun. Pepper.*
				48 *Nutmegs.*
				48 *Cloves.*
		16	192	96 *oun. Saffron.*
				96 *Cinnamon.*

I dare not ſay any thing more of this Rule for fear of running into Algebra upon whoſe frontiers I am already. Upon the ſame account I might very well omit what I deſign to ſay about the two following Rules. But becauſe they are commonly handled in Treatiſes of common Arithmetick, it may not be altogether amiſs to have a touch at them.

Of the Rules of Falſe Poſition.

WHen it happens that the queſtions propoſed are ſuch that you know not under what Rules to bring them, 'tis uſual to attempt their ſolution upon ſuppoſal that certain things taken by peradventure are thoſe you deſired, and afterward trying whether they fulfil the conditions implyed in the queſtion. If they do ſo, 'tis evident they are the things you were in ſearch after. If they prove otherwiſe you frame a proportion, for whoſe ſecond term you put the things ſuppoſed, thoſe you light upon in your tryal of them for the firſt, and the things given for the 3d. And

And if it happen that the 4th. term of the proportion does the buſineſs, the Rule is called the *Rule of ſingle Poſition* or *one falſe Poſition.*

But if this fourth term proves inſufficient, you go on with your work by means of the ſuppoſition already made, and of a new one which you alſo make. And the Rule is then called the *Rule of double Poſition*, or of two Falſe Poſitions. We ſhall explain the latter having firſt laid down ſome examples to facilitate the underſtanding of the former.

(1.) *Example.*

An Army being routed it appears that a ſeventh part were killed upon the ſpot, a third ſaved themſelves by flight, and the 11000 that remained were taken Priſoners, the queſtion is what number the Souldiers were of that made up this Army before the fight, how many dead, and how many eſcaped ?

Muſing a little upon the queſtion, I plainly perceive I muſt have recourſe to the Rule of Three for its ſolution. For $\frac{1}{7}$, $\frac{1}{3}$ and 11000 making up the whole Army, if I take away $\frac{1}{7}$ and $\frac{1}{3}$, *i. e.* $\frac{10}{21}$ from one, for

 there's

there's but one Army, the remainder $\frac{11}{12}$ will be the number of the 11000 Souldiers that escaped. Therefore as $\frac{11}{21}$ parts of the Army are to 11000 Souldiers, so 1 which denotes the whole Army, of which the 11000 Sould. are $\frac{11}{21}$ parts, will be to the number of all the Souldiers that made up the whole Army, which number is 21000.

parts of the Army	*Souldiers*	*Army*	*Souldiers.*
$\frac{11}{21}$	11000 ::	1	21000

But had I not known what Rule to have made use of I must have went this way to work. I make account that a certain number taken by hap hazard is that of all the Souldiers, to ease the work I have chose one whose seventh and 3d. part (which are the parts given) are whole numbers, which is done by taking 21 the product of 7 by 3. If therefore 21 is the number of all the Souldiers, there were but 3 killed, 7 escaped, and the remainder a 11. For 3 the seventh part of 21 and 7 its 3d. part being substracted from 21 leaves 11 for the residue. Now the remainder ought to be 11000. Therefore I put in order a proportion, in which 21 the number of the supposition shall have the second place, the remainder

11

11 found by means thereof the firſt, and the given number 11000 the third, and the fourth term 21000 gives me the number of all the Souldiers, the ſeventh part of whom 3000 were ſlain, the third 7000 eſcape, after which 11000 were made Priſoners.

The remainder found by the ſuppoſition	*the ſuppoſed number.*	*the remainder*	*whole Army.*
11	21 ::	11000	21000

(2.) *Example.*

A man performed the third part of a journey on Horſeback and the fifth part a foot, which betwixt them made up 50 miles. The queſtion is, how many miles the whole journey conſiſted of?

This queſtion alſo may be ſolved by the Rule of Three. For if $\frac{1}{3}$ and $\frac{1}{5}$ or $\frac{8}{15}$ of a journey give 50 miles, 1 or the whole journey will be $93\frac{3}{4}$ ſuch miles.

parts of the journey	*miles* ::	*journey.*	*miles.*
$\frac{1}{3}$ more $\frac{1}{5}$ or $\frac{8}{15}$	50 ::	1	$93\frac{3}{4}$

But if ſo be I cannot tell what Rule to make uſe of in order to its ſolution, I go

about it this way. I suppose a certain number pitched upon by chance is that number of the miles that make the whole journey: that the work may be the easier I pitch upon 15 the product of 3 by 5, to the end that its 3d and fifth part, which are the given parts may be whole numbers. If therefore 15 be the number of all the miles, its 3d. part 5, and 5th. part 3 ought to make 50 miles, now they make only 8. Therefore I put in order a proportion, in which the number found 8 shall have the first place, the number of the supposition 15 the second, and the number given, *viz.* 50 miles the third. After this manner I find the number of miles that make up the whole journey to be $93\frac{3}{4}$, whose third part $31\frac{1}{4}$ was gone on Horseback, and the fifth part $18\frac{3}{4}$ was performed afoot.

Number found	*supposed numb.*	: :	*given numb.*	*miles of the whole journey.*
8	15	: :	50	$93\frac{3}{4}$

(3.) *Example.*

The ages of 3 men make 144 years, the first is three times as old as the second, and the second twice as old as the third. What are the three ages?

Con-

Conſidering the queſtion a little you eaſily perceive that it belongs to the Rule of Fellowſhip. For to ſolve the queſtion you need only divide the given number 144 into three parts that ſhall be proportional to the numbers, 1, 2 and 6 which are ſet, becauſe they expreſs the known relations all the ages bear to each other. For the third of theſe ages is to the ſecond which contains it twice, as 1 to 2 which contains 1 twice, and to the third as 1 to 6, becauſe the ſecond is to the firſt which it contains 3 times, as 2 to 6 which contains 2 thrice. Thus you will find by the Rule of Fellowſhip that the 3 ages are the firſt 72 years, the ſecond 32 years, and the third 16.

$$9 \qquad 144 :: \begin{cases} 6 \\ 2 \\ 1 \end{cases} \qquad \begin{matrix} 72 \\ 32 \\ 96 \end{matrix}$$

But if ſo be I were ignorant of the Rule it belonged to, I go about to ſolve it after this manner. I ſuppoſe the third man is but 1 year old, therefore the thirds age will be 2, and the firſts age 6, and theſe three ages ſhould make 144. Now their ſum is but 9. Therefore I frame a proportion in which the number found 9 ſhall have the

the firſt place, the number of the ſuppoſition which is 1 the ſecond, and the given number the third. The 4th. term of this proportion which is 16 will be the age of the third perſon, the ſeconds age therefore will be twice 16 or 32, and the firſt age 3 times 32 or 96. And theſe three ages make 144.

Number found	*Numb. ſup.*	: :	*the numb. given*	*the leaſt age.*
9	1	: :	144	16

Of the Rule of two Falſe Poſitions.

James Brough

WHen there are ſeveral numbers given that belong to the queſtion, the Rule of ſingle Poſition will not ſerve the turn. For in making uſe of that you muſt write all that's given in the queſtion in the third place of a proportion, and yet you muſt there have but one number. Now in ſuch a caſe as this, you attempt the ſolution thus.

You ſuppoſe a number pitched upon by meer chance is one of thoſe you would have

have, and afterward you try according to the nature of the question whether this number will do or no. If it will not do, observe its excess or want. After that you make choice of a second number instead of that the first supposition helped you to, this also you bring to tryal as you did the former. If this second number proves as ineffectual as the former, you are to observe how much 'tis too big or too little.

The excesses are marked thus +, the wants thus —.

These excesses or wants we will call *errors*, and to the numbers lighted upon by supposition, we shall give no other name than that of *suppositions*.

Rule.

Now to solve the question by means of these suppositions and errours, you clap in the first place of two proportions, the difference of the errours, in the second the difference of the suppositions, and in the third each of the errours: Afterward you add the fourth term of the proportion to that supposition that helped to find it.

You find the difference of the suppositions by substracting the less from the greater, and after the same manner the difference of the errours.

Ex-

Example.

The Ages of three perſons make a 100 years, the firſt age exceeds the ſecond by 24 years, and the ſecond exceeds the third by 20 years, what are the 3 ages?

Let one year be the third and leaſt age, the ſecond therefore will be 21 years, and the firſt 21 more 24 or 45 years, and theſe three ages 1, 21 and 45 ought to make in all 100 years. Now they make but 67. Therefore I frame a proportion whoſe firſt place 67 ſhall occupy, 1 the ſecond, but becauſe there are ſeveral numbers given or known, and you muſt put down but one in the third place, I plainly perceive I cannot come at the ſolution by means of this ſuppoſition alone. Therefore I am content to ſee how much 67 wants of a 100, and I make the want (writing it thus —33) the 1ſt. errour, for 33 from a 100 leaves 67. Afterward I go upon a new ſuppoſition pitching upon 2 years for the third age inſtead of 1 I made choice of before, therefore the ſecond age will be 22 and the firſt 46, and theſe three ages make juſt 70 years. Now they ſhould make a 100, therefore there's a want of 30 years, and I write —30 for the ſecond errour; for 30 from a 100 leaves 70.

This

This being done, I write in the first place of a proportion the difference of the two errours —33 and 30 which is −3, in the second place 1 the difference of the two suppositions 1 and 2, and in the third place each of the errours −33, and —30. The fourth term of the first supposition is 11, and because this term arose from the first supposition 1, which afforded the first errour or want −33, I add 1 to the last term 11 and 12 is the third persons age. Or else the last term of the second proportion is 10, and because this term 10 arises from the second supposition 2, which begat the want −30, I add 2 to 10 and 12 is the age of the 3d. person. The second therefore will be 12 more 20 that is 32 years old, and the first 56, and these three ages 12, 32 and 56 make a 100 years.

1st. supposition 1	*1st. err.* 33	*dif. of suppos.*	*err.*	*3d age*		
2d. —— 2	*2d.* — 30					
differ. of suppos. 1	*dif. of err.* 3	1 : :	$\begin{cases} 33 \\ 30 \end{cases}$	11 *more* 1 (*i. e.* 12	10 *more* 2 (*i. e.* 12	

(2.) *Example.*

The Ages of three persons make a 100 years, the first is 12 years older than the other

other two together, and the ſecond is twice as old as the third, and 8 years over. How old is each perſon ?

Suppoſe 1 year is the third and leaſt Age, the ſecond will therefore be 10, and the firſt 23, the ſum of theſe 3 Ages is 34 years ; Now they ſhould make between them a 100, therefore I write down the want, —66. Suppoſe in the ſecond place the Age of the third perſon to be 2 years, that of the ſecond therefore will be 12, and that of the firſt 26, and the ſum of theſe three Ages is 40 ; But they ſhould make a 100, therefore I put down alſo this want, — 60. This being done I write in the firſt place of a Proportion —6, the difference of the wants — 66 and — 60, and I frame but one of the two proportions, for one will ſerve the turn as well as two ; in the ſecond place I write 1 the difference of the ſuppoſitions, and in the third the want, —66. Its laſt term is 11, to which I add the firſt ſuppoſition 1, which furniſhed the firſt want —66 ; and 12 is the third perſons age, the ſecond therefore will be 2 times 12, and 8 over, *i. e.* 32 years old, and the firſt 12 more 32 and 12, which make 56. And theſe three Ages, 12, 32 and 56, make a 100 years.

Firſt

First ſuppoſi.	1	*firſt errour* 66	*diff. of ſup.*	*firſt err.*	*third age*
Second ſuppoſi.	2	*ſecond err.* 60			
Differ. of ſup.	1	*diff. of err.* 6	1 : :	—66	11mo. 1 1612

(3.) *Example.*

A certain Man meeting a parcel of poor people, and intending to give them 5 farthings a piece, drawing his Purſe he found he had not farthings enough about him by one, and ſo beſtowing upon them but pence a piece, he had 6 farthings left. How many poor people were there, and how many farthings?

Suppoſe 1 to be the number of the poor, now this poor perſon could not receive 5 farthings, becauſe the donor had not ſo many about him by 1, therefore the number of the farthings is but 4. But if he received 4 farthings, the donor muſt have ſix left, (according to another condition of the queſtion) the ſum therefore of the farthings is 4 more 6, *i. e.* 10. Now 4 ſhould be the ſame number with 10, therefore I write down the want, —6. In the ſecond place, ſuppoſe 2 to be the number of the poor, the number of the farthings therefore is twice 5 wanting 1, *viz.* 9. But if each of them received 4 farthings, there ſhould be left 6, the num-

number of the farthings will therefore be 2 times 4 more 6, *i. e.* 14. Now 9 should be the same number as 14, but there wants 5; therefore I write the want —5. These things thus happening, I write in the first place of a proportion —1, the difference of the errours —6 and —5. In the second place I write 1 the difference of the suppositions, and in the third the first errour —6. The last term of this proportion is 6, to which I add the first supposition 1, because it helped me to the first want —6, and thus 7 is the number of the poor people. And there were 34 farthings.

First suppos.	1	*first errour*	6	*diff. of sup.*	*first err.*		*number of the poor.*
Second suppos.	2	*second err.*	5				
Diff. of suppo.	1	*diff. of err.*	1	1	: :	—6	6 *mo.* 1 *is* 7

Under this Rule of *Double Position* fall divers other Examples, of which I shall give the Reader only three, by means of which he will be able without much difficulty to work the rest.

(1.) *Example.*

In *Æsops* time, a Mule and an Asse going on the way together, the latter complained he was overladen, to whom the Mule thus spake: If I should give thee 1 of

of my ſacks, we ſhould have as many one as the other, and if thou ſhouldſt give me 1 of thine, I ſhould have as many again as thee. The Queſtion is, how many ſacks each of them carried?

Suppoſe 1 to be the number of ſacks the Aſſe carried, if ſhe ſhould give one ſack to the Mule ſhe would have never a one left, and the Mule would have twice as many, that is, twice nothing. Since therefore the Mule having one ſack more than its own, has never a one at all, her number of ſacks is —1; If then ſhe ſhould give one ſack to the Aſſe, her number of ſacks would be —2, and the Aſſe would have 2. Now —2 ought to be equal to this number 2, becauſe then they ſhould have as many ſacks one as the other; but 2 exceeds —2 by 4. So I write down the want —4. In the ſecond place, ſuppoſe 2 to be the number of ſacks the Aſſe carried, if ſhe give one ſack to the Mule, ſhe would have one ſtill, and the Mule would have twice as many as ſhe, to wit 2. Since therefore the Mule taking one ſack from the Aſſe has 2, the number of ſacks ſhe had at firſt was 1. If therefore ſhe ſhould give one ſack to the Aſſe, ſhe would have never a one left, and the Aſſe would

would have 3. Now the no ſack that remains to the Mule ſhould be equal to 3; for they ſhould have as many one as the other; but 3 exceeds o by 3, ſo I write that want —3. This being done, I write —1 the difference of the errours or wants —4 and —3 in the firſt place of a proportion, + 1 the difference of the ſuppoſitions in the ſecond, and the firſt want -4 in the third. The fourth term is 4, to which I add the firſt ſuppoſition 1, which cauſed the firſt errour or want -4; And ſo 5 is the number of ſacks the Aſſe carried, and 7 that of the Mules ſacks.

Firſt ſuppo.	1	*firſt errour*	4	*differ. of ſuppoſi.*		*firſt err.*	*the aſſes numb. of ſacks.*
Second ſuppo.	2	*ſecond err.*	3				
Diff. of ſup.	1	*diff. of err.*	1	+1	::	-4	4 *more* 1 *is* 5.

(2.) *Example.*

One gathered in a Garden Apples, Pears and Plums. The number of Plums is 10000, the number of Apples makes half the number of Pears and Plums together, and the number of Pears makes one third of the number of Apples and Plums together. The Queſtion is, how much Fruit the perſon gathered in all, how many Apples and how many Pears?

Sup-

Suppose the number of Pears to be 1, therefore the number of Apples and Plums together will be 3. Now the number of Plums is 10000, the number of the Apples will be therefore 3 −10000, that is −9997, and this number being doubled should make 10000+1 or 10001, to wit the number of Pears and Plums. Now they make only −19994; therefore I write down the want −29995. In the second place, suppose the number of Pears to be 2, the number of Apples and Plums together will therefore be 6, and because the number of Plums is 10000, the number of Apples will be 6 −10000, to wit −9994, and this number being doubled should make 10002 the number of Pears and Plums; now it makes only −19988, therefore I write the default −29990. Lastly, I write −3 the difference of the errours or wants in the first place of a proportion, +1 the difference of the Suppositions in the second place, and the first want −29995 in the third. The last term is 5999, to which I add the first supposition 1, and by this means I know that 6000 is the number of the Pears, 8000 the number of the Apples, and 24000 the number of all the Fruit together the person gathered.

First

Firſt ſuppo.	1	*firſt error*	29995	*diff. of*	*firſt*	*the number of*
Sec. ſuppo.	2	*ſec. error*	29990	*ſuppo.*	*want*	*Pears.*
Diff. of ſup.	1	*dif. of er.*	5	+1 : .	-29995	+5999 *more* (1 *is* 6000.

(3.) *Example.*

A poor man at the point of death ordered by his Will, that the eldeſt of his Children ſhould have 1 Crown out of his whole Eſtate, and the ſeventh part of the remainder, that the ſecond Child ſhould have 2 Crowns, and the ſeventh part of the reſt, the third 3 and the ſeventh part of the remainder, and ſo onward. Now it happens that the diſtribution or diviſion being thus made, they have all a like ſhare. The Queſtion is, how many Children he had, and how much their Father left them?

Suppoſe 1 Crown to be the ſeventh part of the remainder the eldeſt Child was to have, the reſidue therefore will be 7 Crowns, and the value of all the Eſtate the Father had, 8 Crowns, out of which the eldeſt Child having received 1 Crown, there remains 7, of which having got the ſeventh part, to wit 1 Crown, he will have 2 Crowns, and the remainder will be 6 Crowns, out of which the ſecond is to

to have 2 Crowns, after which there remains 4, of which alſo he ought to have a ſeventh part, he therefore will have 2 Crowns and $\frac{4}{7}$; And this number ought to be equal to the 2 Crowns the eldeſt had, but it exceeds it by $\frac{4}{7}$; ſo I ſet this exceſs $\frac{4}{7}$ aſide. In the ſecond place, ſuppoſe 2 Crowns the ſeventh part of the remainder which the eldeſt child ought to have, this remainder therefore will be two times 7 or 14 Crowns, and the Fathers whole Eſtate 15 Crowns, out of which the eldeſt having had 1 Crown there remains 14, of which having alſo received the ſeventh part, he will have 3 Crowns, and the reſidue will be 12, out of which the ſecond child ought to have 2 Crowns, after which there remains 10, of which beſides he ought to have the ſeventh part. He will therefore have 2 Crowns more $\frac{10}{7}$, that is to ſay $3\frac{3}{7}$; And this number ought to be equal to the 3 Crowns the eldeſt child received for his portion; but there is $\frac{3}{7}$ over, therefore I ſet aſide $\frac{3}{7}$ the exceſs. Having done this, I write $\frac{1}{7}$ the difference of the errours or exceſſes in the firſt place of a proportion, 1 the difference of the ſuppoſitions in the ſecond place, the firſt errour, or firſt exceſs $\frac{4}{7}$ in the third; The fourth term is 4, which I add

add to the first supposition 1, which caused the first excesse $\frac{4}{7}$. And so 5 is the 7th. part of the remainder of the poor Mans Estate, which the eldest child ought to have. This remainder will therefore be 7 times 5, to wit 35, to which adding 1 Crown, which he ought to have first of all, the number 36 is the number of Crowns the poor Mans Estate came to, or the whole value of it; out of which the eldest having had 1 more $\frac{35}{7}$, the second 2 more $\frac{28}{7}$, the third 3 more $\frac{21}{7}$, the fourth 4 more $\frac{14}{7}$, the fifth 5 more $\frac{7}{5}$, and finally the last 6 more $\frac{0}{7}$. One shall find that the Father had six Children, to each of which he left 6 Crowns.

1st. supposi.	1	*1st. excess*	$\frac{4}{7}$	*diff. of first*		*the 7th. part of the*
2d. supposi.	2	*2d. excess*	$\frac{3}{7}$	*suppo. excess*		*1st. remainder.*
dif. of sup.	1	*dif. of exc.*	$\frac{1}{7}$	1 ::	$\frac{4}{7}$	4 *mo.* 1 *equal to* 5

The Solution of these three last Questions brings thee (Reader) to the Frontiers of Algebra, 'tis convenient therefore that we stop here, and so take our Repose. Wherefore Reader I take my leave of thee for this time, and bid thee Farewel.

FINIS.